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Interest Rate Spreads in a Theory of Financial Economics: A Proposed Model and Empirical Estimates

by

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ABSTRACT

This essay proposes a method of evaluating the size of the interest rate spread without alluding to any of the common structure-cartel propositions. Instead emphasis is on the component of portfolio risk that banks as financial intermediaries must bear.

Intermediation is taken to be an asset from the point of view of banks. Its acquisition requires that banks maintain a unique portfolio that specifically short-sells deposit instruments so that it can take a position in the loan market that is beyond the limits of its pure equity exposure. The convenient decomposition derived in this essay is that the ensuing portfolio is exposed to the undiversifiable risk that is inherent of loan instruments (lending effect) and that which "borrowing short to lend long" creates (intermediation effect). If such risks have any intrinsic value, it must follow that banks ought to be compensated by a rate of return that appropriately reflects such market valuation. This leads directly into the issue of interest rate spreads since the estimate of the systematic portfolio risk can be used as a reference in determining the size of a risk-related spread.

The model is empirically tested in the case of the Philippines using monthly data for the six-year period between January 1986 to December 1991. The empirical results suggest that the various measures of the actual interest rate spread fall short of the implied "fair" return for undiversifiable risk borne by banks.

In any discussion about financial reform in the Philippines, the issue of the interest rate spread maintained by commercial banks always comes to fore and the subsequent assertion that the commercial banking industry is likely run by a cartel. This is not new and its resurgent nature may be, in part, attributed to the observation that the "empirical" evidence presented to-date has not provided enough basis to finally and unambiguously resolve the issue.

This essay suggests an explanation for the size of the spread without recourse to the usual cartel-related propositions. Instead, the emphasis is on the factors that affect the pricing of financial assets and services through interest rates and the information that these rates actually relay. General tenets of asset pricing theory are used to suggest principally that the discussion over spreads confuses economic with business accounting in an area where the distinction is, in fact, of primary importance.

Section 1 briefly reviews some general material on asset pricing theory, particularly the relationship between risk and return. Section 2 expounds on the applicability of this general framework to banks. The typical commercial bank portfolio is described in terms of the distribution of its individual assets and the undiversifiable risk component of the portfolio is identified using basic tenets of the market model of asset pricing theory. Estimates of risk-consistent returns are then provided and compared with the traditionally cited spreads. Section 3 has some final comments and possible directions for extending the model.

1. The Risk-Return Opportunity Locus

Consider a stylized economy where all investors prefer more returns *ceteris paribus* but are also risk-averse in the process. Assume further that there are only 2 financial assets available, providing random returns equal to Z_{1t} and Z_{2t}. Let ω_1 and ω_2 be the fraction of the portfolio invested in asset 1 and asset 2 respectively where it must be true *a fortiori* that $\omega_1 + \omega_2 = 1$.

If investors were to form a portfolio of these two assets, then the actual return from such a portfolio at time *t* will be equal to:

$$Z_t = \omega_1 Z_{1t} + \omega_2 Z_{2t} \tag{1}$$

with an mean and variance of:[2]

$$\begin{aligned} E(Z) &= w_1\bar{Z}_1 + \omega_2\bar{Z}_2 \\ \bar{Z} &= \bar{Z}_1 + \omega_2(\bar{Z}_2 - \bar{Z}_1) \equiv \mu \end{aligned} \tag{2}$$

$$\begin{aligned} \sigma^2 &\equiv E(Z_t - \bar{Z})^2 \\ &\equiv \omega_1^2\sigma_1^2 + 2\omega_1\omega_2\rho_{12}\sigma_1\sigma_2 + \omega_2^2\sigma_2^2 \end{aligned} \tag{3}$$

where $\bar{Z}_i$ and σ_i^2 respectively denote the average return and the variance of such returns for asset i, i=1,2, while ρ_{12} is the correlation coefficient $\frac{\sigma_{12}}{\sigma_1\sigma_2}$ between asset 1 and asset 2.

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[2]See appendix 1 for the appropriate derivations and extensions.

It is obvious that we can atleast map $0 \leq \omega_2 \leq 1$ into $\sigma_1^2 \leq \sigma^2 \leq \sigma_2^2$. Assume, however, that we only set $\omega_1 + \omega_2 = 1$ but do not require both ω_1 and ω_2 to be positive.[3] Subsequently, equation (2) is a line in (μ,ω_2) space for various values of ω_2 consisent with the adding up condition. As shown in figure 1, μ can always be increased with either ω_1 or ω_2 depending on whether $(\bar{Z}_2 - \bar{Z}_1) \lessgtr 0$.

In contrast, the expression for σ^2 is a nonlinear function in ω_2. Substituting $\omega_1=(1-\omega_2)$, we find that:

$$\sigma^2 = \left[\sigma_2^2 + \sigma_1^2 - 2\rho_{12}\sigma_1\sigma_2\right]\omega_2^2 + 2\left[\rho_{12}\sigma_1\sigma_2 - \sigma_1^2\right]\omega_2 + \sigma_1^2 \tag{4}$$

which is a conic section in (σ^2,ω_2) space, with a vertex at $(\sigma^2=\sigma_1^2,\omega_2=0)$ unless $(\sigma_{12} - \sigma_1^2)$ is extremely negative.[4] In (σ,ω_2) space, this simplifies into a hyperbola as shown in figure 2.

Assuming that Z_t is $\sim N(\mu,\sigma)$, the investor problem can be formalized as maximizing the expected utility of portfolio returns over ω_i:[5]

$$\underset{\omega_i}{\text{Max}}\ E\left[U(Z)\right] = \underset{\omega_i}{\text{Max}}\ E\left[U\left(\mu(\omega_i),\sigma(\omega_i)\right)\right]$$

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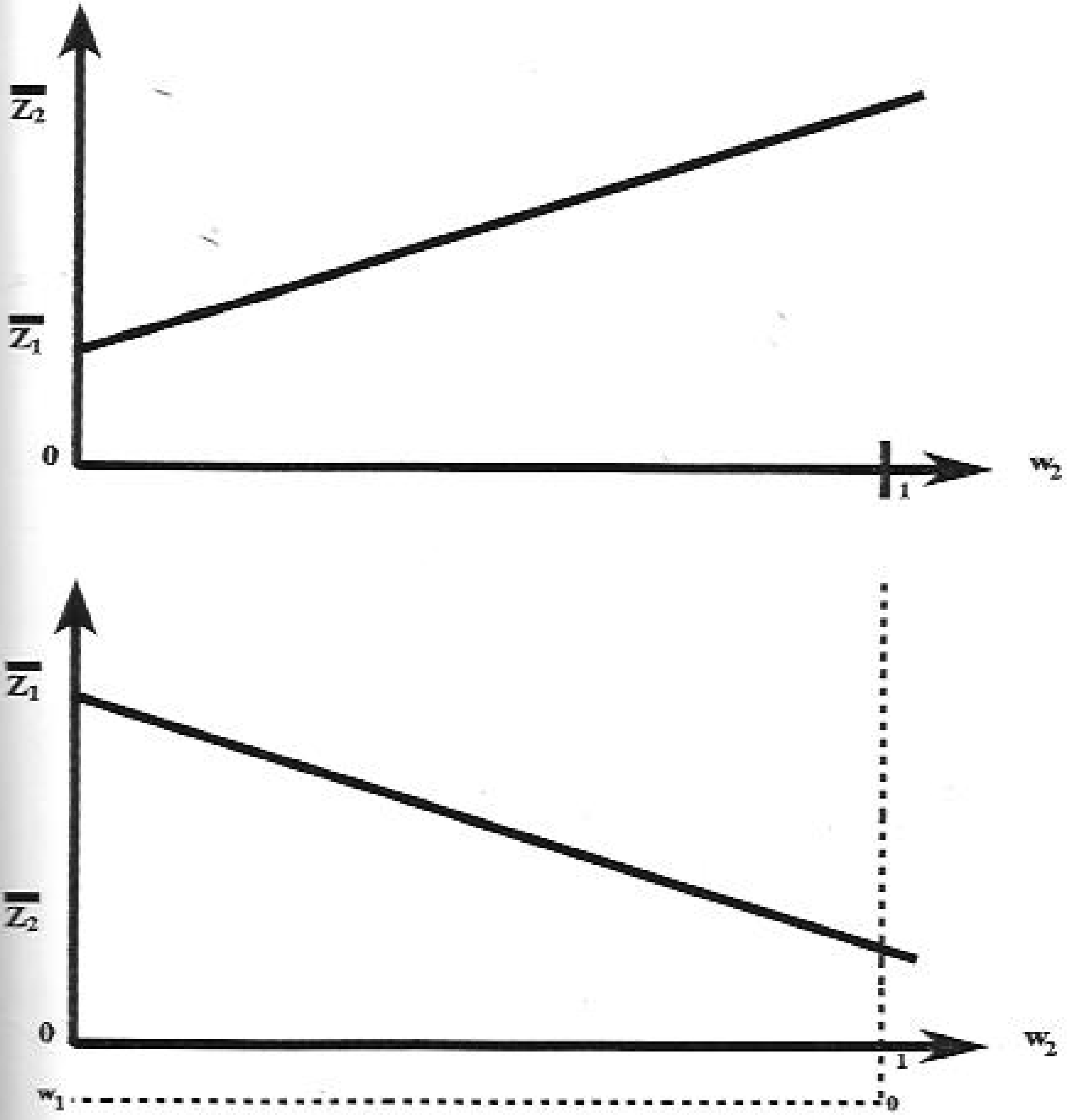
[3]The is consistent with the model proposed by Black (1972). Thus, if $\omega_i < 0$, then ω_j must be greater than unity to satisfy $\omega_i + \omega_j = 1$.

[4]Literally, this suggests that the portfolio cannot be risk-free, i.e. $\sigma = 0$. For a full discussion of the risk-free possibilities when $\rho = \pm 1$, see Ravalo (1991), chapter 4.

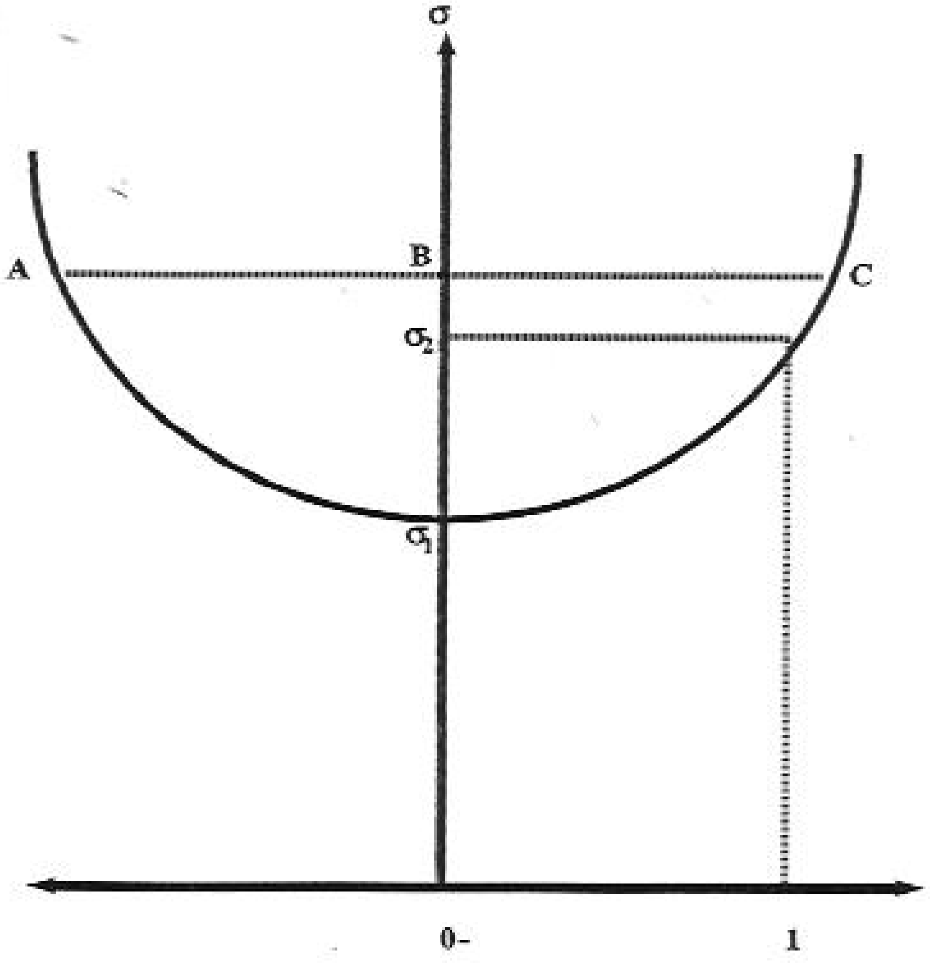
[5]This is Tobin's (1958) "liquidity preference" contribution.

Figure 1



$\overline{Z_2}$
$\overline{Z_1}$
0
1
w_2
$\overline{Z_1}$
$\overline{Z_2}$
0
1
w_2
w_1
0

Figure 2



$$= \underset{\omega_i}{\text{Max}} \int_{-\infty}^{+\infty} U(Z)\ f(Z;\ \mu,\sigma)\ dZ$$

$$= \underset{\omega_i}{\text{Max}} \int_{-\infty}^{+\infty} U(\mu_R+\sigma_R z) \left\{ \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma_R} \exp\left\{ \frac{-1}{2} z^2 \right\} \sigma_R \right\} dz$$

$$= \underset{\omega_i}{\text{Max}} \int_{-\infty}^{+\infty} U(\mu+\sigma z) \left\{ \frac{1}{\sqrt{2\pi}} \exp\left\{ \frac{-1}{2} z^2 \right\} \right\} dz$$

$$= \underset{\omega_i}{\text{Max}} \int_{-\infty}^{+\infty} U(\mu+\sigma z)\ f(z;\ 0,1)\ dz \qquad (5)$$

where $z = \dfrac{Z - \mu}{\sigma}$ is the usual standard normal variable. By implicit differentiation, the indifference curve will be positively sloped:

$$\left.\frac{d\ \mu}{d\ \sigma}\right|_{E(\bar{U})} = - \frac{\int_{-\infty}^{+\infty} z\ U'(Z)\ f(z;0,1)\ dz}{\int_{-\infty}^{+\infty} U'(Z)\ f(z;0,1)\ dz} \quad > \quad 0 \qquad (6)$$

and convex upwards, $\dfrac{d\ \mu^2}{d^2\ \sigma} > 0$, since $\int_{-\infty}^{+\infty} z\ U'(Z)\ f(z;0,1)\ dz$ is equal to:

$$\left[\int_{-\infty}^{0} z\ U'(\mu_R+\sigma_R z)\ f(z)\ dz \quad + \quad \int_{0}^{+\infty} z\ U'(\mu_R+\sigma_R z)\ f(z)\ dz\right] \quad < \quad 0 \qquad (7)$$

for all risk-averse investors nonsatiated with Z_t.[6]

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[6] See for example Ravalo (1991) chapter 3 for a derivation.

Investors of this type would then strictly prefer the concave set σ_1AB over the convex set σ_1CB in figure 3 since a higher μ is obtained at the same σ. For the same reason, figure 4 shows that investors cannot do better than the frontier σ_1DA and therefore take this as the relevant domain in making their portfolio choice.[7]

With the opportunity frontier σ_1DA defined, the risk-averse investor now faces a portfolio trade-off. Higher returns can be acquired by increasing ω_i, for $\overline{Z}_i > \overline{Z}_j$ $i \neq j$. However, for as long as the returns of the ith asset is positively correlated with Z_t, $\sigma_{ip} > 0$, such strategy can be shown to increase risk since $\frac{d\sigma^2}{d\omega_i} > 0$ by equation (3).[8]

Without loss of generality, assume that asset 2 is riskier than asset 1. Subsequently, $\sigma_2^2 > \sigma_1^2$ and $\overline{Z}_2 > \overline{Z}_1$. Explore now the possibility that the portfolio risk, defined in equations (3) & (4), will be higher than the risk implied by asset 2. Thus,:

$$\omega_1^2\sigma_1^2 + 2\omega_1\omega_2\rho_{12}\sigma_1\sigma_2 + \omega_2^2\sigma_2^2 \quad > \quad \sigma_2^2$$
$$(1-\omega_2)^2\sigma_1^2 + 2(1-\omega_2)\omega_2\rho_{12}\sigma_1\sigma_2 \quad > \quad (1-\omega_2^2)\sigma_2^2$$

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[7]See Hirschleifer (1964) and Merton (1972) for general discussions on the shape of this opportunity set.

[8]See appendix 2 for a formal proof. Note, however, that the positive correlation condition makes sense *a priori* because one would obviously be interested to invest in an asset which tends to drive up the portfolio return as reflected in figure 1.

Figure 3

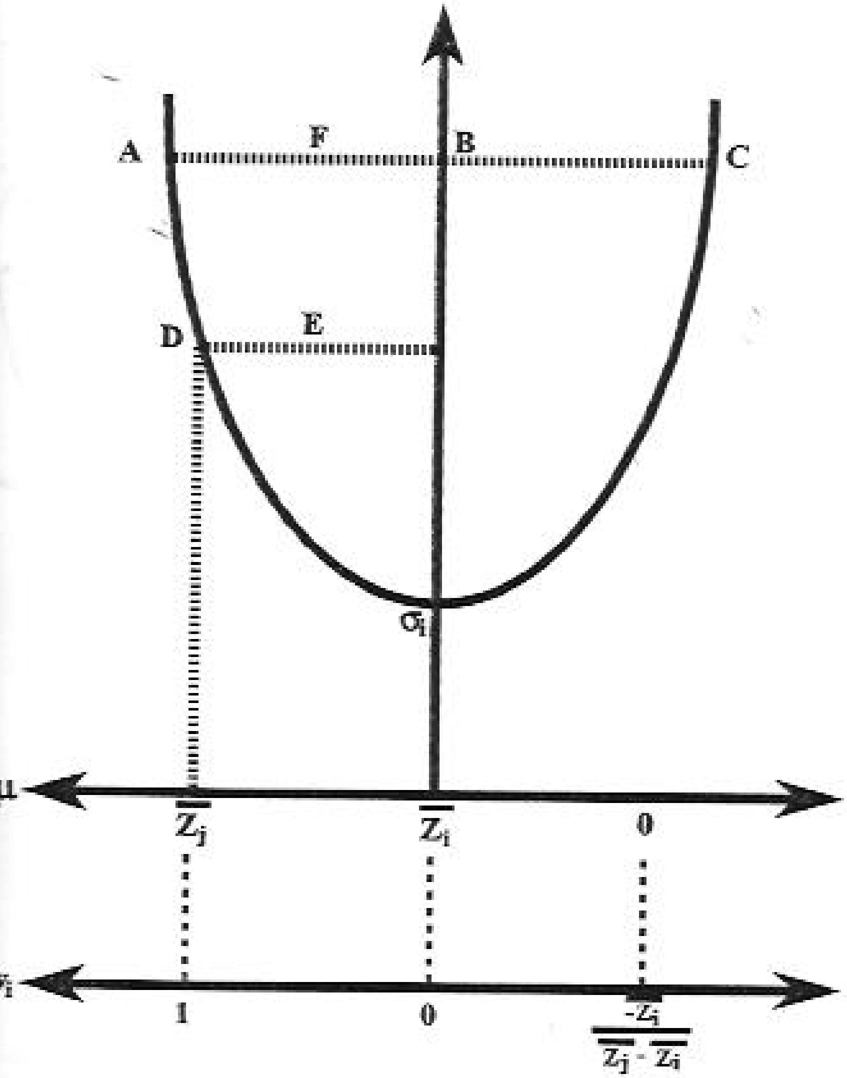
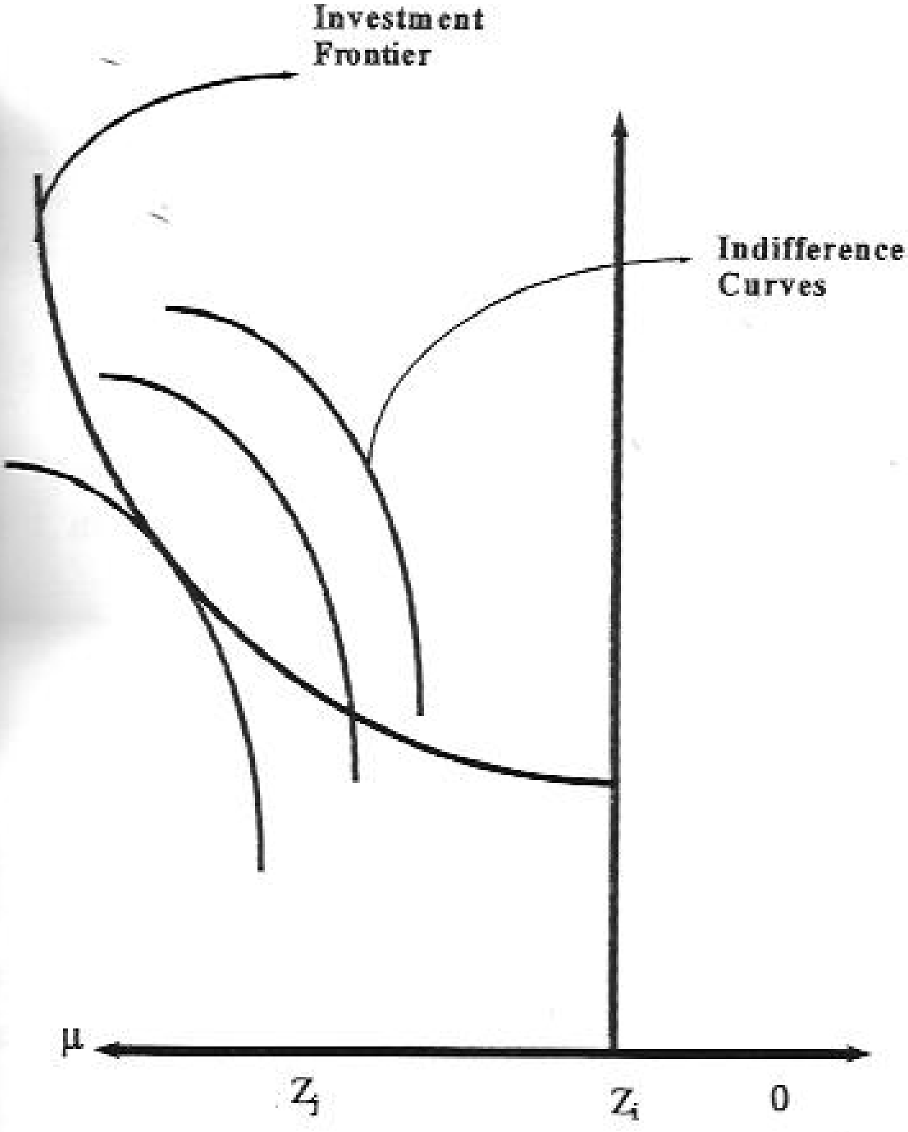


Figure 4



$$1 + 2\rho_{12}\frac{\omega_2\sigma_2}{\omega_1\sigma_1} > \left(\frac{1+\omega_2}{1-\omega_2}\right)\left(\frac{\sigma_2}{\sigma_1}\right)^2 \tag{8}$$

The choice variable is again ω_2 although ρ_{12} remains unspecified. If $0 < \omega_2 < 1$, then,

$$\left(\frac{1+\omega_2}{1-\omega_2}\right)\left(\frac{\sigma_2}{\sigma_1}\right)^2 > 1 \quad \Rightarrow \quad 2\rho_{12}\frac{\omega_2\sigma_2}{\omega_1\sigma_1} > 0 \tag{9}$$

which reduces to $\rho_{12} > 0$.[9] If, on the other hand, the portfolio is such that $\omega_2 > 1$, then,

$$\left(\frac{1+\omega_2}{1-\omega_2}\right)\left(\frac{\sigma_2}{\sigma_1}\right)^2 < -1 \quad \Rightarrow \quad 2\rho_{12}\frac{\omega_2\sigma_2}{\omega_1\sigma_1} > -2 \tag{10}$$

Since $\sigma_2 > \sigma_1$ *a priori*, this suggests that $\rho_{12} < 1$ is at least a necessary condition for the portfolio to be riskier than asset 2.[10] Evidently, then, positive proportions of asset 2 in the portfolio will make the latter riskier than the former, conditional only on $\rho_{12} \neq 1$.

Two key elements are worth re-stating. First, we re-emphasized that investors who are risk-averse and nonsatiated with returns face a concave frontier

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[9]This includes the polar case of $\rho = 1$.

[10]This includes $-1 \leq \rho \leq 0$ which satisfies equation (8) *a fortiori*.

of investment alternatives beyond which opportunities with higher returns given the same level of risk or the same expected rate of return at lower levels of risk are infeasible. Second, which point on the frontier the investor will choose depends on ω_i which in turn determines not only the portfolio mix and but also the (μ,σ) characteristic of the chosen portfolio. We showed in particular that specific combinations of positive valued ω_i and ρ_{ij} will make a composite portfolio riskier than the highest risk ith asset.

2. An Application to Commercial Banks[11]

If the bank's portfolio is a necessary consequence of operations, then "intermediation" can be taken as a form of composite asset which only banks hold. If we can further relate investment in intermediation to particular values of ω_i, we can then get a sense of the rate of return that is consistent with the finite opportunity frontier. This is merely an application of the separation theorem which states that the efficient frontier will be common to all investors regardless of differences in their degree of risk-aversion.[12]

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[11]We assume that a two-moment model is sufficient to fully describe the portfolio. Feldstein (1969) shows that two-moment models are exact when the utility function is quadratic or when the random deviation of actual from expected returns is distributed normally. See also Epps (1981) for a relevant discussion.

[12]Cass and Stiglitz (1970) discuss this point at length, providing a framework that relates risk tolerance to terminal wealth. This can be tied directly to banks since banks in general are assumed to be maximizers of terminal wealth. See Santomero (1984) for the most recent survey of bank models.

2.1 The Portfolio Structure of Commercial Banks

If banks lent out equity funds exclusively, they would not be any different from money-lenders, defined fully by:

$$L_0 = W_0 \tag{11}$$

where L_0 denotes "basic" loans and W_0 is equity. Banks, however, are structurally different because they are precisely the only financial institutions allowed to simultaneously source deposits and extend credit, alleviating the funding constraint by increasing investibles to:

$$W_0 + D = A \tag{12}$$

where D and A are deposits and total assets respectively.

The asset base is then an "enhanced" loan portfolio $\tilde{L} = W_0 + D > L_0$ which is possible only by borrowing deposits. Subsequently, the bank's portfolio is defined uniquely by joint holdings of loans and deposits:

$$\tilde{L} + D = \omega_2 W_0 + \omega_1 W_0 = W_0 \tag{13}$$

taking a "long" position on loans by short selling deposits with equity serving as the margin requirement.[13] Since:

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[13]For an excellent introductory discussion of short selling under various conditions, see Elton and Gruber (1987) chapters 2 & 3. Dyl (1975) modifies the

$$\frac{\tilde{L}}{W_0} = 1 + \frac{D}{W_0} = 1+\lambda \tag{14}$$

this is further exacerbated by the extremely high degree of leverage, λ, which these institutions maintain.[14]

This would imply that $\omega_1 < 0$ and $\omega_2 > 1$. If we let deposits be asset 1 and enhanced loans be asset 2, then the discussion in section 1 suggests that the bank's portfolio is in fact riskier than the pure loan portfolio of a money-lender. In determining the risk content of the former, the latter then offers a convenient lower-bound from which we can deduce the rate of return that banks expect as risk-remuneration for simultaneously sourcing deposits and extending credit.

2.2 The Valuation of Risk

The risk content of loans can be properly derived under an asset pricing model. The most fundamental of the various models of asset pricing theory (APT) is Sharpe's (1963) diagonal model which postulates a direct relationship between the returns of an asset and the return of the market. Based on this, we can

Black model to include margin requirements for short selling (i.e. requiring the investor to put up part of the funds needed to purchase short) and shows that the new frontier is to the left and above of Black's frontier, essentially having a vertex at the origin.

[14]International leverage ratios of 15:1 or higher are more common than they are rare. See July 6, 1992 issue of *Business Week* for a survey of the world's top 200 banks.

express the respective returns of loans and deposits as:[15]

$$r_t = \alpha_L + \beta_L R_{mt} + \varepsilon_{Lt} \tag{15}$$

$$i_t = \alpha_D + \beta_D R_{mt} + \varepsilon_{Dt} \tag{16}$$

where R_{mt} is the return on the market proxy at time t, α_L & α_D reflect the components of the return on loans and deposits that are independent of market performance and the ε's are white noise.

The key parameters are the "beta" estimates $\hat{\beta}_L$ and $\hat{\beta}_D$ because $\beta_i = \frac{d\,\sigma}{d\,\omega_i}$ and is therefore a measure of risk.[16] Since the portfolio beta is just the weighted sum of the betas of the individual assets that comprise the portfolio,[17]

$$\beta = \omega_1\beta_D + \omega_2\beta_L \tag{17}$$

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[15]In this form, this is called the market model.

[16]See appendix 2 for the derivation. Also, note that the general formulation of the market model, $R_{i,t} = \alpha + \beta R_{m,t} + \varepsilon_t$, implies that:

$$\sigma^2_{R_i} = \beta^2\sigma^2_{R_m} + \sigma^2_{\varepsilon}$$

The exact definition of undiversifiable (systematic) risk, $\beta^2\sigma^2_{R_m}$, is thus a multiple of β (with $\sigma^2_{R_m}$ taken as a constant) and it is in this sense that this parameter continues to receive prominent attention.

[17]See appendix 3 for a succinct proof.

this can be re-stated in terms of the estimates $\hat{\beta}_L$ and $\hat{\beta}_D$ such that:

$$\hat{\beta} = \hat{\beta}_L - \omega_1(\hat{\beta}_L - \hat{\beta}_D) \tag{18}$$

Since intermediation brings about a portfolio that is unique to banks, it can then be taken as an asset held exclusively by bank investors. In this sense $\hat{\beta}$ is the rate of return on "intermediation" that compensates for exposure to risks. But since intermediation involves both the sourcing of deposits and the provision of loans, $\hat{\beta}$ is therefore also an estimate of a "fair" spread that is due to pure systematic risk. If $\hat{\beta} \geq 1$, then we expect the portfolio to carry a spread that would at least be equal to R_{mt} since the beta of the market portfolio is equal to unity. If, however, $\hat{\beta} < 1$, then the spread would be bounded by R_{mt} from above and by r from below since $\hat{\beta} > \hat{\beta}_L$ for $\omega_1 < 0$.[18]

2.3 Preliminary Estimates Using All-Maturity Rates

Monthly interest rates for time deposits, secured loans and the average money market rate, WAIR, for the 6 year period January 1986 - December 1991 were gathered from the Central Bank Center for Statistical Information (CBCSI). To

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[18]The traditional methodology of analyzing the market structure of banks via the size of the interest rate spread must implicitly require a well-defined competitive banking market, comparable and known *a priori*, to be used as a reference case. Without such a comparable competitive market, market structure theory does not allow a determination whether a spread of k% is "high" or "low" in absolute terms. Direct application of this traditional approach to the Philippine case causes disturbing--though overlooked--theoretical and practical difficulties.

convert these to a fixed-base real series, monthly inflation was calculated from the consumer price index (CPI) for the NCR region as published by the National Statistical Coordination Board (NSCB). Using January 1986 as the base period, the real rates were computed as:[19]

$$R = \frac{N - \pi}{1 + \pi} \tag{19}$$

where R, N represent the real rate and the nominal rate respectively and π is the percentage change of the price index with respect to the fixed base period.

Estimates for equations (15) and (16) based on the average time deposit rate and the average secured loan rate are listed in table 1 using 60 of the available 72 data points.[20] The results suggest that the interest rate spread that should have accrued to banks as compensation for bearing undiversifiable risk should historically have more than approximated the WAIR in real terms. Such statement is primarily significant

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[19]This is taken from the exact definition of:

$$1 + R = \frac{1 + N}{1 + \pi}$$

In general, the approximation $R = N - \pi$ should only be made when the accumulated intertemporal price change is insignificant. Note as well that π measures accumulated price changes in terms of a fixed base rather than the commonly reported monthly inflation rates. The latter is the price change relative to the same month in the previous year and thus its use implicitly propagates the oversight of using a moving base. Such practice clearly has no theoretical basis.

[20]These coefficients--and all those that follow--have been adjusted for first degree autocorrelation.

because the nominal spread that is prescribed by the results as risk-compensating is greater than the commonly cited spreads already labelled by most as excessive and perceived to be a consequence of "the" cartel. By way of illustration, the nominal spread implied by the OLS estimates is listed in table 2 for the years 1990-91.[21]

It turns out however that the results are questionable for atleast three reasons. First, note the possibility that the difference between $\hat{\beta}_L=0.9733$ and $\hat{\beta}_D=0.9357$ may be due to sampling error and may not be statistically significant. In particular, consider two assets which we assume *a priori* to satisfy:

$$\overline{Z_1} = \overline{Z_2} = m \tag{20}$$

$$\sigma_1^2 = \sigma_2^2 = v^2 \tag{21}$$

Direct application of equations (2) and (3) will verify that any composite portfolio of these two assets for all pairs of ω_1 and ω_2 that satisfy the adding up condition will be defined by:

$$\bar{Z} = \omega_1\overline{Z_1} + \omega_2\overline{Z_2} = m \tag{22}$$

$$\sigma^2 = \sum_{i=1}^{2}\sum_{j=1}^{2} \omega_i\omega_j\sigma_{ij} = v^2\left\{1 - 2(1-\rho_{12})(1-\omega_1)\omega_1\right\} \tag{23}$$

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[21]It is important to note that while the spread in column 6 is more often cited, it is subject to very serious biases. First, savings deposits can be essentially withdrawn on call while loan instruments are bound by a specific term and as such the difference in their rates must include this difference in liquidity *ipso facto*. Second, this spread does not adjust for the costs attributable to imposed regulations.

Thus, within the context of the two-parameter model of asset pricing, if $\beta_L = \beta_D$ then the two assets are identical with respect to their systematic risk component and should provide identical expected (market valued) rates, $\overline{Z_1} = \overline{Z_2}$. As evident from equation (23), there are still diversification possibilities that can be exploited, $\sigma^2 < v^2$, even when $\overline{Z_1} = \overline{Z_2}$ but only under the condition that $\rho_{12} \neq 1$ and $\omega_1 < 1$. But expressions (20) and (21) are in fact tantamount to $\rho_{12} = 1$ if these are to remain limited to and consistent with the two-parameter model. Hence, the composite portfolio formed cannot be any different from the risk-return characteristic of either asset since $\sigma^2 = v^2$ when $\rho_{12} = 1$.

The consequences of $\hat{\beta}_L = \hat{\beta}_D$ therefore transcend pure econometrics. The bank would have absolutely no incentive to facilitate intermediation if $\sigma^2 = v^2$ *a priori*. Therefore, it must expect to find $\sigma^2 < v^2$, atleast under some condition that it has control over, which in turn allows us to expect $\hat{\beta}_L > \hat{\beta}_D$. Although $\hat{\beta}_L = \hat{\beta}_D$ simplifies equation (18), this has the bigger impact of reducing banks to the level of money-lenders since intermediation has no estimated market value in terms of risk.

To test the hypothesis of across-equation equality, the model was estimated simultaneously using the seemingly unrelated regression model:

$$
\begin{aligned}
r_t &= \alpha_L + \beta_L R_{mt} + \varepsilon_{Lt} \\
i_t &= \alpha_D + \beta_D R_{mt} + \varepsilon_{Dt} \\
&H_O: \beta_L - \beta_D = 0 \\
&H_A: \beta_L - \beta_D \neq 0
\end{aligned}
\qquad (24)
$$

As a system, the portfolio beta attributable to "pure" loans is estimated at $\hat{\beta}_L=0.964$ while the marginal effect of short sales has a beta factor of $\hat{\beta}_L-\hat{\beta}_D=0.02138$. As is evident from table 3, the Wald statistic suggests that the data does not reject the null hypothesis, both at the 5% and 10% level of significance since the critical chi-squared statistic at 1 degree of freedom is 3.84 and 2.71 respectively.

Second, note that both the theory and the various possible estimation methods assume that the distribution of returns is known *a priori* and has remained stationary. As Barry (1978) shows, there are important measurement errors to consider when these assumptions are not satisfied. It is reasonable to believe in our case that the data does not satisfy the latter and thus, at the very least diminish the reliability of the estimates. Towards this end, note that the *ex post* forecasts in table 1 generally underestimated the actual 1991 data. Other technical concerns exist and in fact forms of estimation bias--particularly errors in variables and the omitted variables problem--continue to hound the empirical literature of single-index models. At the very least, these should be considered in evaluating $\hat{\beta}$.

2.4 An Issue of Aggregation, Convenience and Bias

The third--and perhaps the most crucial--source of bias arises from the use of all-maturity loan and deposit rates. If all interest rates are random draws from the same population, there may be little harm done. It is less clear, however, when financial instruments are not perfectly substitutable and the fund market is effectively segmented.

All-maturity rates can be represented as dot products $r=\mathbf{yr}$ and $i=\mathbf{xi}$ where y and x are vectors of weights and **r** and **i** are vectors whose elements indicate the respective rates in the different maturity categories defined by the CBCSI. It follows therefore that:

$$\hat{\beta}_L = \frac{\text{Cov}(r,\text{R}_m)}{\text{Var}(\text{R}_m)} = \frac{\sigma_{r,\text{R}_m}}{\sigma^2_{\text{R}_m}} = \frac{\text{Cov}(\mathbf{yr},\text{R}_m)}{\text{Var}(\text{R}_m)} \tag{25}$$

where
$$\begin{aligned}
\sigma_{r,\text{R}_m} &= \sum_t \left\{ \sum_k y_k \mathbf{r}_{kt} - \sum_k y_k \overline{\mathbf{r}_{kt}} \right\} \left\{ \text{R}_{mt} - \overline{\text{R}_{mt}} \right\} \\
&= \sum_t \left\{ \sum_k y_k \left[\mathbf{r}_{kt} - \overline{\mathbf{r}_{kt}} \right] \right\} \left\{ \text{R}_{mt} - \overline{\text{R}_{mt}} \right\} \\
&= \sum_t \left\{ y_1(\mathbf{r}_1 - \overline{\mathbf{r}_1})(\text{R}_m - \overline{\text{R}_m}) + y_2(\mathbf{r}_2 - \overline{\mathbf{r}_2})(\text{R}_m - \overline{\text{R}_m}) + \dots \right\} \\
&= y_1 \sum_t (\mathbf{r}_1 - \overline{\mathbf{r}_1})(\text{R}_m - \overline{\text{R}_m}) + y_2 \sum_t (\mathbf{r}_2 - \overline{\mathbf{r}_2})(\text{R}_m - \overline{\text{R}_m}) + \dots \\
&= y_1 \sigma_{r_1 R_m} + y_2 \sigma_{r_2 R_m} + y_3 \sigma_{r_3 R_m} + \dots \\
&= \sum_k y_k \sigma_{r_k R_m}
\end{aligned} \tag{26}$$

As a result, it now turns out that because $r=\mathbf{yr}$ and $i=\mathbf{xi}$, then:

$$\hat{\beta}_L = \frac{\sum_k y_k \sigma_{r_k R_m}}{\sigma^2_{R_m}} = y_1\hat{\beta}_{1L} + y_2\hat{\beta}_{2L} + y_3\hat{\beta}_{3L} + \ldots \tag{27}$$

$$\hat{\beta}_D = \frac{\sum_k x_k \sigma_{i_k R_m}}{\sigma^2_{R_m}} = x_1\hat{\beta}_{1D} + x_2\hat{\beta}_{2D} + x_3\hat{\beta}_{3D} + \ldots \tag{28}$$

$$\hat{\beta}_L - \hat{\beta}_D = (y_1\hat{\beta}_{1L} - x_1\hat{\beta}_{1D}) + (y_2\hat{\beta}_{2L} - x_2\hat{\beta}_{2D}) + \ldots \tag{29}$$

where $\hat{\beta}_{kL} = \frac{\sigma_{r_k R_m}}{\sigma^2_{R_m}}$ and $\hat{\beta}_{kD} = \frac{\sigma_{i_k R_m}}{\sigma^2_{R_m}}$ are the OLS estimates of a regression of r_k and i_k respectively on R_m for the k<u>th</u> maturity category.

By definition, $\sum_k y_k = \sum_k x_k = 1$. If, however, $y_k = x_k \ \forall \ k$ (i.e. $\mathbf{y}=\mathbf{x}$), equation (29) simplifies into:

$$\hat{\beta}_L - \hat{\beta}_D = y_1(\hat{\beta}_{1L} - \hat{\beta}_{1D}) + y_2(\hat{\beta}_{2L} - \hat{\beta}_{2D}) + \ldots \tag{30}$$

where:

$$\hat{\beta}_{kL} - \hat{\beta}_{kD} = \frac{\sigma_{r_k R_m}}{\sigma^2_{R_m}} - \frac{\sigma_{i_k R_m}}{\sigma^2_{R_m}}$$

$$= \frac{\sum_t (r_{kt} - \overline{r_{kt}})(R_m - \overline{R_m})}{\sigma^2_{R_m}} - \frac{\sum_t (i_{kt} - \overline{i_{kt}})(R_m - \overline{R_m})}{\sigma^2_{R_m}}$$

$$= \frac{\sum_t \left\{(r_{kt} - \overline{r_{kt}}) - (i_{kt} - \overline{i_{kt}})\right\} (R_m - \overline{R_m})}{\sigma^2_{R_m}}$$

$$= \frac{\sum_t \left\{(r_{kt} - i_{kt}) - (\overline{r_{kt}} - \overline{i_{kt}})\right\} (R_m - \overline{R_m})}{\sigma^2_{R_m}} \tag{31}$$

But as argued previously, intermediation services, *I*, can be treated as an asset which can intuitively be defined as having an expected return $\mu_{I,t} = (\overline{r_t} - \overline{i_t})$ and variance $\sigma^2_I = \sigma^2_r + \sigma^2_i - 2\sigma_{ri}$. Then:

$$\hat{\beta}_{kL} - \hat{\beta}_{kD} = \frac{\sigma_{I_k R_m}}{\sigma^2_{R_m}} = \hat{\beta}_k \quad \forall\ k \tag{32}$$

$$\hat{\beta}_L - \hat{\beta}_D = \sum_k y_k \hat{\beta}_k = \sum_k y_k \left\{\hat{\beta}_{kL} - \hat{\beta}_{kD}\right\} \tag{33}$$

The parameter $\hat{\beta}_k$ is critical because it measures the degree of sys-tematic r isk that intermediation in the kth category bears. As such, the point of $\hat{\beta}_L - \hat{\beta}_D$ is eloquently defined by equation (33).

Unfortunately, there is no particular reason why **y**=**x**. Consequently, estimates of $\hat{\beta}_L - \hat{\beta}_D$ that use all-maturity rates are not for equation (33) but rather equation (29) where not only is $\hat{\beta}_{kL} - \hat{\beta}_{kD}$ important but also the relative magnitudes of y_k and x_k $\forall$ k. This also implies that the across-equation constraint in the SURE model tests for:

$$(y_1\hat{\beta}_{1L} - x_1\hat{\beta}_{1D}) + (y_2\hat{\beta}_{2L} - x_2\hat{\beta}_{2D}) + \ldots + (y_n\hat{\beta}_{nL} - x_n\hat{\beta}_{nD}) = 0 \qquad (34)$$

rather than:

$$y_1\left\{\hat{\beta}_{1L} - \hat{\beta}_{1D}\right\} + y_2\left\{\hat{\beta}_{2L} - \hat{\beta}_{2D}\right\} + \ldots + y_n\left\{\hat{\beta}_{nL} - \hat{\beta}_{nD}\right\} = 0 \qquad (35)$$

While $\left\{\hat{\beta}_{kL} - \hat{\beta}_{kD}\right\} = \hat{\beta}_k$ has an explicit meaning, it is not intuitively obvious what $(y_k\hat{\beta}_{kL} - x_k\hat{\beta}_{kD})$ is supposed to reflect and subsequently how expression (34) is to be interpreted.

The existence of non-overlapping maturity categories also raises a further issue. The notion of the yield curve points out that instruments with longer terms generally carry a premium, loosely specified as:

$$x_k = f(\tau_k, \ldots) + \varepsilon_k \qquad \text{where } \frac{\partial x_k}{\partial \tau_k} > 0; \quad E(\varepsilon_k) = 0 \qquad (36)$$

where τ_k is a unit of time of length k. Having distinct categories:

$$\begin{aligned} \chi_1 &= f^1(\tau_1, \ldots) & + \varepsilon_1 \\ \chi_2 &= f^2(\tau_2, \ldots) & + \varepsilon_2 \\ &\;\vdots \\ \chi_{n-1} &= f^{n-1}(\tau_{n-1}, \ldots) & + \varepsilon_{n-1} \\ \chi_n &= f^n(\tau_n, \ldots) & + \varepsilon_n \end{aligned} \tag{37}$$

requires that $\tau_n > \tau_{n-1} > \ldots > \tau_2 > \tau_1$ which implies, *ceteris paribus*:

$$E(\chi_n) > E(\chi_{n-1}) > \ldots > E(\chi_2) > E(\chi_1) \tag{38}$$

Assuming for simplicity that all arguments in $f^k(\cdot)$ are fixed and that $\varepsilon_k \sim N(0,\sigma_k)$, it follows that $\chi_k \sim N(\mu_k,\sigma_k)$. The average for all *n* maturity categories will then be:

$$\mu_\chi = \omega_1\mu_1 + \omega_2\mu_2 + \omega_3\mu_3 + \ldots + \omega_n\mu_n = \sum_k^n \omega_k\mu_k \qquad ;\sum_k^n \omega_k = 1 \tag{39}$$

with a variance of:

$$\sigma_\chi^2 = \sum_i^n \omega_i^2\sigma_i^2 + \sum_i^n \sum_{\substack{j \\ j\neq i}}^n \omega_{ij}\rho_{ij}\sigma_i\sigma_j \tag{40}$$

where ρ_{ij} is the correlation coefficient between categories i and j.

The question of whether the different observed rates are draws from the same population is a problem of ANOVA where we seek to test:

$$\begin{array}{l} H_0: \{\mu_1 = \mu_2 = \mu_3 = \ldots = \mu_n\} \\ H_A: \{\mu_1 \neq \mu_2 \neq \mu_3 \neq \ldots \neq \mu_n\} \end{array} \tag{41}$$

H_0 is rejected if differences among the class means, $\mu_{k-1}-\mu_k \; \forall \; k$, are large and/or when within class variances, $\sigma_k^2 \; \forall \; k$, are also large. Hence, to incorrectly assume a single population is to use an average μ_χ that broadly deviates from (1) the class means and from (2) the actual sample. If the point of using the average is that it is an implicit proxy for the rate on the "representative" instrument, then the error of assuming H_0 instead of H_A is definitely costly because the descriptive power of μ_χ is weakened by these unwarranted variations. In contrast, the converse is at little cost since μ_k & σ_k^2, will not be very different across classes, with σ_χ^2 relatively small and μ_χ a fairly compact summary of the class means $\mu_k \; \forall \; k$.

The obvious convenience of using scalar (i.e. average) rates may therefore be at a significant cost. Fortunately, much of what is needed to extend equations (15) and (16) to account for several types of loan and deposit instruments has already been introduced.

Consider specifically equation (17) again. In general:

$$\beta = \sum_k^n y_k \beta_{kL} + \sum_k^n x_k \beta_{kD} \tag{42}$$

where there are n forms of loan and deposit instruments while y_k and x_k are weights which satisfy the following conditions:

(i) $x_k \leq 0 \quad \forall\ k=1...n;$

(ii) $y_k > 0 \quad \forall\ k=1...n;$

(iii) $\sum_k^n y_k + \sum_k^n x_k = 1$

Risk-averse bank investors are likely to distribute equity funds over all n categories. Define δ_k as the proportion of W_0 in the k<u>th</u> category such that $\sum_k^n \delta_k = 1$. Assume further that sourced deposits are used to extend credit in the same category. It follows then that:

$$\left\{y_k + x_k > 0\right\} = \delta_k \quad \forall\ k \tag{43}$$

which can be re-arranged as:

$$y_k = \delta_k - x_k \quad \forall\ k=1...n \tag{44}$$

Substituting this back into equation (42), we get:

$$\beta = \sum_k^n \left(y_k\beta_{kL} + x_k\beta_{kD}\right)$$

$$= \sum_{k}^{n} \left[(\delta_k - x_k)\beta_{kL} + x_k\beta_{kD} \right]$$

$$= \sum_{k}^{n} \delta_k\beta_{kL} - \sum_{k}^{n} x_k(\beta_{kL} - \beta_{kD}) \qquad (45)$$

Equation (45) is a generalization of equation (18) and a restatement of equation (29). When short sales are not allowed, banks are pure money-lenders and its portfolio risk will reflect the weighted average risk of all loan instruments, $\sum_{k}^{n} \delta_k\beta_{kL}$. With short sales, there is added risk from (1) having borrowed funds and (2) managing such funds in the pursuit of arbitrage profits. The term $(\beta_{kL} - \beta_{kD})$ reflects such increment and can be thought of as the pure risk of intermediation. Subsequently, the second term in (45) is the weighted "price" of market risk attributable to selling various deposit instruments short.

The special nature of banking institutions comes from the management of uncertainty and it is this feature of short selling that is *the* crux of bank operation. As Porter (1967) argues, the tenet of profit maximization in micro theory implies:

"...that the bank should acquire a portfolio consisting entirely of the asset whose yield (less any cost of maintenance and acquisition) is greatest. But this procedure misses the very essence of banking, which is to 'borrow short and lend long'."

This has the effect of creating a market value for financial intermediation and nothing short of this will be appropriate for a "bank".

2.5 Estimates Using *n* Maturity Categories

The model was consequently re-estimated by OLS as:

$$
\begin{aligned}
r_{1t} &= \alpha_{1L} + \beta_{1L}R_{mt} + \varepsilon_{1Lt} \\
r_{2t} &= \alpha_{2L} + \beta_{2L}R_{mt} + \varepsilon_{2Lt} \\
r_{3t} &= \alpha_{3L} + \beta_{3L}R_{mt} + \varepsilon_{3Lt} \\
r_{4t} &= \alpha_{4L} + \beta_{4L}R_{mt} + \varepsilon_{4Lt} \\
r_{5t} &= \alpha_{5L} + \beta_{5L}R_{mt} + \varepsilon_{5Lt} \\
r_{6t} &= \alpha_{6L} + \beta_{6L}R_{mt} + \varepsilon_{6Lt} \\
\\
i_{1t} &= \alpha_{1D} + \beta_{1D}R_{mt} + \varepsilon_{1Dt} \\
i_{2t} &= \alpha_{2D} + \beta_{2D}R_{mt} + \varepsilon_{2Dt} \\
i_{3t} &= \alpha_{3D} + \beta_{3D}R_{mt} + \varepsilon_{3Dt} \\
i_{4t} &= \alpha_{4D} + \beta_{4D}R_{mt} + \varepsilon_{4Dt} \\
i_{5t} &= \alpha_{5D} + \beta_{5D}R_{mt} + \varepsilon_{5Dt} \\
i_{6t} &= \alpha_{6D} + \beta_{6D}R_{mt} + \varepsilon_{6Dt}
\end{aligned}
\qquad (46)
$$

using data from by the CBCSI.[22] The complete results are in appendix 4.

Table 4 suggests that the correlation with the market proxy is more diverse than implied by the initial estimates $\hat{\beta}_L = 0.97$ and $\hat{\beta}_D = 0.93$. In column 5, the price of intermediation diverges substantially from the 0.04 average. Nowhere is this more clear than with $\hat{\beta}_{1l}=0.13$ which would have been higher if not for the

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[22]Note that time deposits have data for the "30-45 day" and the "46-60 day" categories while the loan rates only go as far as "less than 60 days". To provide some comparison, a simple average of these two time deposit categories was also calculated. Subsequent estimates are provided for the average as well as for the independent components.

apparent dominance of 46-60 day TDs.

To test for across-equation constraints, the model was again re-estimated as SURE with the coefficients estimated by GLS.[23] Two types of hypotheses were then tested using the Wald statistic: (1) that intermediation jointly bears no risk:

$$\left\{\hat{\beta}_{1I} = \hat{\beta}_{2I} = \hat{\beta}_{3I} = \hat{\beta}_{4I} = \hat{\beta}_{5I} = \hat{\beta}_{6I}\right\} = 0 \tag{47}$$

and (2) that the same is independently true for each category:

$$\hat{\beta}_{kI} = \hat{\beta}_{kL} - \hat{\beta}_{kD} = 0 \quad \forall\ k=1,2,3,4,5,6 \tag{48}$$

In table 5, we reject the hypothesis that $\beta_{kI} = \beta_{kL} - \beta_{kD}$ is jointly zero for all 6 maturity categories. This is in direct contrast to the results obtained earlier with all-maturity rates where we failed to reject the hypothesis of equality between $\hat{\beta}_L$ and $\hat{\beta}_D$. On an individual category basis, the same conclusion is obtained. This is significant because these rates are for secured loans and in effect is implicit evidence that intermediation still involves undiversifiable risk despite the collateral. This would seem to imply that either banks continue to rely on interest rates to reflect the necessary market signals and/or that there is difficulty--or atleast some negative preference--in having collateral fully cover the pertinent risk exposure.[24]

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[23]The model was corrected for autocorrelation, assumed to be:

$$\varepsilon_{i,t} = \rho_i \varepsilon_{i,t-1} + u_{i,t} \quad \forall\ i=1,2,3,4,5,6 \text{ and } \forall\ t$$

The only exception is category 4 (6-12 month instruments) where the data cannot reject the null hypothesis. Interestingly, the β_{kI}'s in the other categories fall within a "high" range 0.059-0.168--higher than the 0.04 average implied previously--in contrast to the much "lower" 0.016 value estimated for category 4.

Ceteris paribus, one would expect that β_L, β_D and $\beta_L - \beta_D$ would all rise with longer maturities since the inherent risk of deposit pretermination against loan defaults would be more emphasized. The GLS estimates above, however, show practically no such pattern and if at all, β_{kI} appears to generally decline as the term increases. A possible explanation for this may be the common practice of roll-overs. Depositors seem to prefer to simulate long term instruments by rolling-over shorter term accounts at the expense of lower rates to gain better flexibility and liquidity. With banks releasing long term credit in tranches that are periodically subject to review, the bank may now be operating a riskier short term market because the threat of mismatch between pretermination and defaults under a short-sale financed portfolio is much more evident with the roll-overs. This is exacerbated by the fact that roll-overs artificially creates a larger more volatile volume of short term transactions which may be an added source of risk.

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[25]This is in fact circular reasoning. If the collateral offered fully covered the bank's exposure and is fairly "liquid", then the bank becomes totally indifferent to defaults since it can always benefit from the proceeds of the collateral. But if such collateral exists, then the client would not have to borrow in the first place. Thus, the usefulness of the collateral, both as a signal and as insurance, is only when its value is less than the worth of the loan contract or when there are significant costs to be avoided in converting the collateral to cash.

3. Final Comments and Further Directions

This essay proposes a method of evaluating the size of the interest rate spread without alluding to any of the common structure-cartel propositions but instead emphasizes the component of portfolio risk that banks as intermediaries must bear. Intermediation is taken to be an asset from the point of view of banks and its acquisition requires that banks maintain a unique portfolio that short-sells deposit instruments so that it can take a position in the loan market that is beyond the limits of its pure equity exposure. The convenient decomposition derived in this essay is that the ensuing portfolio is exposed to the un-diversifiable risk that is inherent of loan instruments (lending effect) and that which "borrowing short to lend long" creates (intermediation effect). If such risks have any intrinsic value, it must follow that banks ought to be compensated by a rate of return that appropriately reflects such market valuation. This leads directly into the issue of interest rate spreads since the estimate of the systematic portfolio risk can be used as a reference in determining the size of a risk-related spread. The empirical results suggest that the various measures of actual spread fall short of the level that is implied as a "fair" return to undiversifiable risk borne by banks.

The advantage of this approach is two fold. First, it provides for a theoretically-supported framework for evaluating the actual magnitude of the spread. Given the appropriate data, it is feasible to actually determine if a particular spread is absolutely high or low. Second, the model is clearly time-variant in that the estimates change when the basis of comparison, the systematic risk implied by the market proxy, changes. This is particularly critical because it allows for intangible changing market conditions to be factored into the pricing framework. Such, after all, is the economic essence of interest rates as a signaling mechanism of market conditions.

The model, however, is susceptible to possible technical caveats. Clearly, the estimates assume that asset returns have a stationary normal distribution. Matters easily become very complex when the distribution is not stationary and much worse, if it is unknown. In the literature, Bayesian estimates, for example, have been proposed--specifically predictive distributions--to handle the problem of unknown and nonstationary distribution of asset returns. There is also outside evidence that results may be sensitive to the type of test used. Using SURE to test simultaneous nonlinear restrictions on the intercept of a combined CAPM-market model, Gibbons (1982) used the likelihood ratio test to reject the CAPM. Interestingly, Stambaugh (1982) reaches a very different conclusion after using the same estimation method but with the Langrangian Multiplier test instead. Amidst all these, the usual discussion over specification bias continues to elicit active work. If anything, this only shows that much more work is needed before a definitive conclusion can be made or atleast that the results will eventually become an empirical issue on a case to case basis.

The Roll (1977) critique of the empirical literature of the CAPM argues that the numerical estimates will be sensitive to the chosen market proxy. If numerical estimates for comparison with actual spreads are the end in view, one may chose to run the GLS model using different proxies for the market portfolio since the relationship between risk and return is itself theoretically robust despite the known estimation difficulties. In general, it seems acceptable to propose that there is an inherent relationship between return and risk, particularly in response to dynamic macroeconomic conditions.[26]

The discussion over interest rate spreads has, indeed, created a lot of noise and will continue to do so because of its sensitive role in describing the competitiveness of our commercial bank market. Such noise is however compounded by grave technical errors that linger but generally remain uncited. The empirical evidence reported here suggest that the actual spread between the all-maturity rates is far lower than what the OLS model would imply (table 2 column 3 vs column 5). When compared to the more often cited (though theoretically incorrect) spread between the weighted secured loan rate and the savings deposit rate, the same conclusion is obtained.

There is likewise an unknowing technical error in the continued insistence of using convenient scalars (i.e. average rates). As shown by the derivations, particularly of equations (29), (33) and (45), these scalar indices do not generally reflect the intended information and instead are much more vulnerable to further "noise". A comparison of the empirical results of the OLS and GLS models bears this out since very divergent conclusions are drawn with respect to the component of undiversifiable risk that banks face as a result of intermediating between savers and dissavers. Both numerically and analytically, it should be very evident that much more signals can be read from the simultaneous system GLS model rather than the simple OLS model.

Further work abounds, particularly in the area of estimation. If forecasts

[25]See Francis and Fabozzi (1979) for evidence that β responds to the business cycle.

are to be desired, more effort must be made towards stationarity. Various market proxies can be tested to insure the robustness of the results and the common estimation biases well cited in the single-index model literature can be directly addressed. With the proper software that can handle inequality constraints, it would also be of interest to pursue the hypothesis that β_{kl} changes with maturity, within a generalized model that could account for non-price schemes.

The issue about the size of the spread is far from closed. While the empirical results suggest that the actual spread--however defined--is generally lower than the implied market-determined "fair" level, that is not likely to go unchallenged. That is admittedly a significant departure, if one is to be made at all, from the repetitive and now common practice of implying non-competitive behavior in our financial markets. At the very least, the suggested framework above reminds us that some caution must be exercised before a final and categorical statement can be made about the size of the interest rate spread.

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TABLE 1
OLS Estimates for Secured Loans and Time Deposits

$$r = \underset{(8.116)}{3.0554} + \underset{(32.29)}{0.9733}\ R_{mt}$$

Adjusted R^2 = 0.9687
F(1,58) = 1829.5

Log-Likelihood Ratio	=	208.954
Akaike Information	=	1.503
DW (Untransformed)	=	1.378
Amemiya Prediction	=	4.496
DW (Transformed)	=	1.904

	Actual	Forecast	Diff		Actual	Forecast	Diff
61	-21.847	-26.106	4.259	67	-32.039	-30.066	-2.973
62	-25.900	-27.719	1.819	68	-32.593	-29.505	-3.088
63	-27.438	-26.615	-0.823	69	-33.282	-30.906	-2.376
64	-28.900	-28.634	-0.266	70	-32.128	-31.304	-0.824
65	-29.546	-27.950	-1.596	71	-33.910	-30.989	-2.921
66	-31.360	-30.580	-0.780	72	-33.090	-32.462	-0.628

$$i = \underset{(0.109)}{0.0471} + \underset{(27.62)}{0.9357}\ R_{mt}$$

Adjusted R^2 = 0.97200
F(1,58) = 2049.14

Log-Likelihood Ratio	=	215.559
Akaike Information	=	1.348
DW (Untransformed)	=	1.052
Amemiya Prediction	=	3.849
DW (Transformed)	=	1.940

	Actual	Forecast	Diff		Actual	Forecast	Diff
61	-24.862	-27.987	3.125	67	-34.487	-31.795	-2.692
62	-28.036	-29.539	0.577	68	-35.149	-31.256	-3.893
63	-28.962	-28.477	-0.485	69	-37.693	-32.602	-4.071
64	-30.145	-30.418	0.273	70	-35.514	-32.985	-2.529
65	-31.772	-29.760	-2.012	71	-35.795	-32.682	-3.113
66	-33.312	-32.289	-1.023	72	-35.794	-34.064	-1.730

Note: The values in parenthesis under the estimated coefficients are the t-values after correcting for AR(1).

TABLE 2
Real and Nominal Spreads Implied by the Regression Estimates

Period (1)	Real Spread (2)	Nominal Spread (3)	Inc. Due to ω_1 (4)	Actual Spreads I (5)	II (6)
1990:01	-16.9769	14.9947	0.3761	3.035	16.554
:02	-16.5322	15.9593	0.3805	4.367	17.132
:03	-15.7811	17.6188	0.3903	3.967	18.541
:04	-15.9115	20.3088	0.4221	3.159	19.770
:05	-17.0917	19.6317	0.4336	3.285	19.683
:06	-21.7417	15.0647	0.4582	1.632	16.478
:07	-23.2839	15.7707	0.4952	4.497	18.791
:08	-20.7317	20.1382	0.5036	4.106	16.853
:09	-21.1919	21.5985	0.5305	3.638	20.860
:10	-23.8842	17.9186	0.5351	3.968	20.466
:11	-25.4109	17.3935	0.5587	4.059	21.256
:12	-20.6451	29.9063	0.6242	3.264	22.753
1991:01	-29.1521	18.2466	0.6505	5.032	27.150
:02	-30.7653	16.5109	0.6632	3.594	21.182
:03	-29.6609	19.6855	0.6824	2.747	18.792
:04	-31.6799	17.2496	0.6955	2.137	17.431
:05	-30.9959	19.0969	0.7056	3.842	15.817
:06	-33.6249	16.3409	0.7304	3.420	14.373
:07	-33.1111	18.1659	0.7444	4.325	16.334
:08	-32.5507	21.0391	0.7723	4.588	17.116
:09	-33.9509	21.8031	0.8203	8.135	17.998
:10	-34.3488	20.2329	0.8075	6.202	20.135
:11	-34.0336	21.7403	0.8216	3.479	13.126
:12	-35.4707	19.4553	0.8262	5.007	17.784

Notes:

Column 2: = (0.973)(Real WAIR)
= Real spread attributable to pure systematic risk

Column 3: = (0.973)(Real WAIR)$(1+\pi)$ + π
= Nominal spread implied by degree of systematic risk when loanable funds are sourced purely from equity

Column 4: = (0.037)(Real WAIR)$(1+\pi)$ + π
= Increase in nominal spread attributable to short selling deposits to be able to increase loan portfolio beyond equity

Column 5: = All-maturity loan rate - All-maturity time deposit rate

Column 6: = All-maturity loan rate - Savings deposit rate

TABLE 3
Constrained Regression Model
Generalized Least Squares

A. Unconstrained Estimates

Estimates for equation: Secured Loan Rate

Observations	= 60		
Mean of LHS	= -0.574	Std.Dev of LHS	= 11.7982
StdDev of residuals	= 1.9807	Sum of squares	= 227.5643
R-squared	= 0.9713	Adj. R-squared	= 0.9708
Durbin-Watson Stat.	= 1.8674	Autocorrelation	= 0.0662
RHO used for GLS	= 0.3094		

Variable	Coeff.	Std. Error	t-ratio	Prob|t|≥x	Mean	Std.Dev.
Constant	3.0023	0.3844	7.810	0.00000		
RWAIR	0.96495	0.03058	31.552	0.00000	-3.7461	11.799

Estimates for equation: Time Deposit Rate

Observations	= 60		
Mean of LHS	= -3.4051	Std.Dev of LHS	= 11.5335
StdDev of residuals	= 1.7271	Sum of squares	= 173.0114
R-squared	= 0.9771	Adj. R-squared	= 0.9768
Durbin-Watson Stat.	= 1.7131	Autocorrelation	= 0.1434
RHO used for GLS	= 0.4229		

Variable	Coeff.	Std. Error	t-ratio	Prob|t|≥x	Mean	Std.Dev.
Constant	0.05778	0.3987	0.145	0.88475		
RWAIR	0.94357	0.0312	30.191	0.00000	-3.7461	11.799

NOTE: Estimates have been corrected for AR(1)

TABLE 3 (continued)
Constrained Regression Model
Generalized Least Squares

B. Constrained Estimates:

Estimates for equation: Secured Loan Rate

Observations	= 60		
Mean of LHS	= -0.5742	Std.Dev of LHS	= 11.7982
StdDev of residuals	= 1.9840	Sum of squares	= 228.3149
R-squared	= 0.9712	Adj. R-squared	= 0.9707
Durbin-Watson Stat.	= 1.8397	Autocorrelation	= 0.0801
RHO used for GLS	= 0.3094		

Wald test: $\chi^2(1) = 1.8474$, Probability = 0.17409

Variable	Coeff.	Std. Error	t-ratio	Prob|t|≥x	Mean	Std.Dev.
Constant	2.9629	0.3833	7.730	0.00000		
RWAIR	0.9560	0.0298	32.003	0.00000	-3.7461	11.799

Estimates for equation: Time Deposit Rate

Observations	= 60		
Mean of LHS	= -3.4051	Std.Dev of LHS	= 11.5335
StdDev of residuals	= 1.7302	Sum of squares	= 173.6337
R-squared	= 0.9771	Adj. R-squared	= 0.9767
Durbin-Watson Stat.	= 1.7439	Autocorrelation	= 0.1280
RHO used for GLS	= 0.4229		

Wald test: $\chi^2(1) = 1.8474$, Probability = 0.17409

Variable	Coeff.	Std. Error	t-ratio	Prob|t|≥x	Mean	Std.Dev.
Constant	0.0984	0.3975	0.248	0.80446		
RWAIR	0.95605	0.0298	32.003	0.00000	-3.7461	11.799

NOTE: Estimates have been corrected for AR(1)

TABLE 4
OLS Beta Estimates For Various Maturity Categories

Category	Term of Instrument	Estimated β_k Secured Loans	Estimated β_k Time Deposits	Pure Risk Effect of Intermediation
A	30 - 45 Days		0.28008	
B	46 - 60 Days		0.88094	
1	< 60 Days	0.99765	0.86383	0.13382
2	61 - 90 Days	0.98346	0.99690	-0.01344
3	91 - 180 Days	0.95918	0.87867	0.08051
4	181 - 365 Days	0.95149	0.94585	0.00564
5	365 - 730 Days	1.16762	1.00158	0.16604
6	> 730 Days	1.01546	0.94925	0.15621
7	All Maturities	0.97326	0.93567	0.03759

TABLE 5
GLS Beta Estimates
Test of Across Equation Constraints

Category	$\hat{\beta}_{kL}$	$\hat{\beta}_{kD}$	$\hat{\beta}_{kI}$	Std. Error	t-Ratio	Prob $\|t\| \geq x$ $\chi^2(1)$
1	0.98770	0.87462	0.113080	0.01898	5.957	0.00000
2	0.98083	0.88412	0.096715	0.01932	5.005	0.00000
3	0.95267	0.87108	0.081599	0.02169	3.761	0.00017
4	0.95912	0.94229	0.016830	0.03470	0.485	0.62767
5	1.16340	0.99470	0.168700	0.06307	2.675	0.00747
6	1.01180	0.95226	0.059495	0.02856	2.083	0.03725

APPENDIX 1
General N-Asset Model

If there are instead n assets in the economy, the expected portfolio return will therefore be:

$$E\left[Z\right] = \bar{Z} = E\left[\sum_{i=1}^{n} \omega_i Z_i\right] = \sum_{i=1}^{n} \omega_i \bar{Z}_i \qquad \textit{Result 1}$$

The riskiness of such a portfolio can then be intuitively measured in units that reflect the deviation of actual returns from the expected return in result (1). If we define

σ_i^2 = variance of the return of asset i = $E(Z_i-\bar{Z}_i)^2$

σ_{ij} = covariance between assets i & j = $E\left[(Z_i-\bar{Z}_i)(Z_j-\bar{Z}_j)\right]$

σ^2 = variance of the returns of the portfolio = $E(Z-\bar{Z})^2$

it is easy to show that σ^2 can be expressed as:

$$\sigma^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \omega_i \omega_j \, \sigma_{ij} \qquad \textit{Result 2}$$

Proof:

$$\sigma^2 = E(Z-\bar{Z})^2$$

$$= \mathrm{E}\left(\sum_{i=1}^{n} \omega_i Z_i - \sum_{i=1}^{n} \omega_i \bar{Z}_i\right)^2$$

$$= \mathrm{E}\left(\sum_{i=1}^{n} \omega_i \ (Z_i - \bar{Z}_i)\right)^2$$

$$= \mathrm{E}\left(\left(\sum_{i=1}^{n} \omega_i \ (Z_i - \bar{Z}_i)\right)\left(\sum_{i=1}^{n} \omega_i \ (Z_i - \bar{Z}_i)\right)\right)$$

$$= \mathrm{E}\left(\sum_{i=1}^{n} \sum_{j=1}^{n} \omega_i \omega_j (Z_i - \bar{Z}_i)(Z_j - \bar{Z}_j)\right)$$

$$= \left(\sum_{i=1}^{n} \sum_{j=1}^{n} \omega_i \omega_j \ \mathrm{E}\left((Z_i - \bar{Z}_i)(Z_j - \bar{Z}_j)\right)\right)$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} \omega_i \omega_j \ \sigma_{ij} \qquad \blacksquare$$

APPENDIX 2

Since: $$\sigma^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} \omega_i\omega_j\,\sigma_{ij} = \sum_{i=1}^{n} \omega_i^2\sigma_i^2 + \sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} \omega_i\omega_j\,\sigma_{ij}$$

then: $$\frac{d\,\sigma^2}{d\,\omega_i} = \frac{d}{d\,\omega_i}\left\{\sum_{i=1}^{n} \omega_i^2\sigma_i^2 + \sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} \omega_i\omega_j\,\sigma_{ij}\right\}$$

$$= 2\omega_i\sigma_i^2 + 2\sum_{\substack{j=1\\ j\neq i}}^{n} \omega_j\,\sigma_{ij}$$

$$= 2\left\{\omega_i\sigma_i^2 + \omega_1\sigma_{i1} + \omega_2\sigma_{i2} + \omega_3\sigma_{i3} + \ldots + \omega_n\sigma_{in}\right\}$$

$$= 2\left\{\omega_1\sigma_{i1} + \omega_2\sigma_{i2} + \omega_3\sigma_{i3} + \ldots + \omega_i\sigma_i^2 + \ldots + \omega_n\sigma_{in}\right\}$$

Note however that:

$$\sigma_{i1} = E\left[(Z_i - \bar{Z}_i)(Z_1 - \bar{Z}_1)\right]; \quad \sigma_{i2} = E\left[(Z_i - \bar{Z}_i)(Z_2 - \bar{Z}_2)\right]$$

$$\sigma_{i3} = E\left[(Z_i - \bar{Z}_i)\ (Z_3 - \bar{Z}_3)\right]; \ \ldots; \sigma_{in} = E\left[(Z_i - \bar{Z}_i)\ (Z_n - \bar{Z}_n)\right]$$

and so:

$$\sum_{j=1}^{n} \omega_j\sigma_{ij} = \left\{\omega_1\sigma_{i1} + \omega_2\sigma_{i2} + \omega_3\sigma_{i3} + \ldots + \omega_i\sigma_i^2 + \ldots + \omega_n\sigma_{in}\right\}$$

$$= \omega_1 E\left[(Z_i - \bar{Z}_i)(Z_1 - \bar{Z}_1)\right] + \omega_2 E\left[(Z_i - \bar{Z}_i)(Z_2 - \bar{Z}_2)\right] + \ldots$$

$$= E\left[\omega_1(Z_i - \bar{Z}_i)(Z_1 - \bar{Z}_1) + \omega_2(Z_i - \bar{Z}_i)(Z_2 - \bar{Z}_2) + \ldots\right]$$

$$= E\left[\left(Z_i - \bar{Z}_i\right)\left(\omega_1(Z_1 - \bar{Z}_1) + \omega_2(Z_2 - \bar{Z}_2) + \ldots\right)\right]$$

$$= E\left[\left(Z_i - \bar{Z}_i\right)\left(\omega_1 Z_1 + \ldots + \omega_n Z_n - \omega_1\bar{Z}_1 - \ldots - \omega_n\bar{Z}_n\right)\right]$$

$$= E\left[\left(Z_i - \bar{Z}_i\right)\left(Z - \bar{Z}\right)\right]$$

$$= \sigma_{ip}$$

Hence: $\dfrac{d\,\sigma^2}{d\,\omega_i} = 2\sigma_{ip} = 2\rho_{ip}\sigma_i\sigma_p$ ▮ *Result 3*

where σ_{ip} and ρ_{ip} are respectively the covariance and correlation coefficient between the ith asset and the portfolio.

Similarly,:

$$\sigma = \left\{\sum_{i=1}^{n} \omega_i^2\sigma_i^2 + \sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} \omega_i\omega_j\,\sigma_{ij}\right\}^{\frac{1}{2}}$$

$$\frac{d\,\sigma}{d\,\omega_i} = \frac{d}{d\,\omega_i}\left\{\sum_{i=1}^{n} \omega_i^2\sigma_i^2 + \sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} \omega_i\omega_j\,\sigma_{ij}\right\}^{\frac{1}{2}}$$

$$\frac{d\,\sigma}{d\,\omega_i} = \frac{\frac{1}{2}\left\{2\omega_i\sigma_i^2 + 2\sum_{\substack{j=1\\ j\neq i}}^{n}\omega_j\sigma_{ij}\right\}}{\left\{\sum_{i=1}^{n}\omega_i^2\sigma_i^2 + \sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\omega_i\omega_j\,\sigma_{ij}\right\}^{\frac{1}{2}}}$$

$$= \frac{\left\{\omega_i\sigma_i^2 + \sum_{\substack{j=1\\ j\neq i}}^{n}\omega_j\sigma_{ij}\right\}}{\sigma_p}$$

$$= \frac{\sigma_{ip}}{\sigma_p} \qquad \textit{Result 4}$$

If the particular portfolio in question is the "market portfolio", then:

$$\frac{d\,\sigma}{d\,\omega_i} = \frac{\sigma_{im}}{\sigma_m} = \sigma_m\hat{\beta} \qquad \textit{Result 5}$$

where $\hat{\beta}$ is the least squares estimate in a regression of the form:

$$R_{it} = \alpha + \beta R_{mt} + \varepsilon_{it} \qquad \textit{Result 6}$$

where R_{it}, R_{mt} are returns of the i<u>th</u> security and the market portfolio respectively in period t and ε is the random error term. This is exactly the same β that is at the core of asset pricing theory.

APPENDIX 3
Portfolio Beta vs. Security Beta

Recall that by definition:

$$\beta_i = \frac{\text{Covariance (Asset i \& Market Portfolio M)}}{\text{Variance of M}} = \frac{\sigma_{iM}}{\sigma_{MM}}$$

Hence:

$$\omega_i\beta_i = \omega_i\frac{\sigma_{iM}}{\sigma_{MM}} = \frac{\omega_i\sigma_{iM}}{\sigma_{MM}}$$

$$\sum_{i=1}^{n}\omega_i\beta_i = \sum_{i=1}^{n}\frac{\omega_i\sigma_{iM}}{\sigma_{MM}}$$

$$= \frac{\sum_{i=1}^{n}\omega_i\sigma_{iM}}{\sigma_{MM}}$$

$$= \frac{1}{\sigma_{MM}}\sum_{i=1}^{n}\omega_i\sigma_{iM}$$

$$= \frac{\sigma_{PM}}{\sigma_{MM}}$$

$$= \beta_P \quad \blacksquare$$

Result 7

APPENDIX 4
OLS Regression Results For Various Maturity Categories

Note: Values in parenthesis under the estimated coefficients are the t-values after correcting for AR(1).

$$r_1 = \underset{(9.244)}{3.3056} + \underset{(33.98)}{0.9976}\ R_{mt}$$

Adjusted R^2 = 0.9648
F(1,58) = 1619.57

Log-Likelihood Ratio	=	201.879			
Akaike Information	=	1.6639	Amemiya Prediction	=	5.2803
DW (Untransformed)	=	1.5999	DW (Transformed)	=	1.8903

$$r_2 = \underset{(7.16)}{2.8854} + \underset{(30.65)}{0.9835}\ R_{mt}$$

Adjusted R^2 = 0.9693
F(1,58) = 1865.50

Log-Likelihood Ratio	=	210.087			
Akaike Information	=	1.5136	Amemiya Prediction	=	4.5432
DW (Untransformed)	=	1.2788	DW (Transformed)	=	1.8606

$$r_3 = \underset{(6.474)}{2.8166} + \underset{(27.54)}{0.9592}\ R_{mt}$$

Adjusted R^2 = 0.9582
F(1,58) = 1352.35

Log-Likelihood Ratio	=	191.4693			
Akaike Information	=	1.7795	Amemiya Prediction	=	5.9268
DW (Untransformed)	=	1.3696	DW (Transformed)	=	1.9528

$$r_4 = \underset{(5.823)}{3.293} + \underset{(21.42)}{0.9515}\ R_{mt}$$

Adjusted R^2 = 0.9510
F(1,58) = 1146.5021

Log-Likelihood Ratio	=	182.0027			
Akaike Information	=	1.9689	Amemiya Prediction	=	7.1631
DW (Untransformed)	=	1.1099	DW (Transformed)	=	2.0347

$$r_5 = \underset{(7.878)}{6.080} + \underset{(19.03)}{1.1676}\ R_{mt}$$

Adjusted R^2 = 0.9238
F(1,58) = 716.6324

Log-Likelihood Ratio	=	155.5167			
Akaike Information	=	2.8004	Amemiya Prediction	=	16.4516
DW (Untransformed)	=	1.2755	DW (Transformed)	=	2.2005

$$r_6 = \underset{(8.57)}{3.9262} + \underset{(27.77)}{1.1055}\ R_{mt}$$

Adjusted R^2 = 0.9614
F(1,58) = 1469.1613

Log-Likelihood Ratio	=	196.243			
Akaike Information	=	1.817	Amemiya Prediction	=	6.1563
DW (Untransformed)	=	1.3194	DW (Transformed)	=	1.9175

$$i_A = \underset{(-0.44)}{-4.644} + \underset{(3.92)}{0.2800}\ R_{mt}$$

Adjusted R^2 = 0.9628
F(1,58) = 1526.2476

Log-Likelihood Ratio	=	198.4453			
Akaike Information	=	1.5086	Amemiya Prediction	=	4.5207
DW (Untransformed)	=	0.0262	DW (Transformed)	=	1.7137

$$i_B = \underset{(-4.85)}{-1.655} + \underset{(31.89)}{0.8758} \; R_{mt}$$

Adjusted R^2 = 0.9647
$F(1,58)$ = 1615.0621

Log-Likelihood Ratio	=	201.7181			
Akaike Information	=	1.4057	Amemiya Prediction	=	4.0784
DW (Untransformed)	=	1.4185	DW (Transformed)	=	1.7872

$$i_1 = \underset{(-3.74)}{-1.623} + \underset{(25.47)}{0.8638} \; R_{mt}$$

Adjusted R^2 = 0.9682
$F(1,58)$ = 1795.8346

Log-Likelihood Ratio	=	207.8741			
Akaike Information	=	1.3229	Amemiya Prediction	=	3.7544
DW (Untransformed)	=	1.0341	DW (Transformed)	=	1.9046

$$i_2 = \underset{(-4.94)}{-1.707} + \underset{(32.24)}{0.8869} \; R_{mt}$$

Adjusted R^2 = 0.9718
$F(1,58)$ = 2034.6702

Log-Likelihood Ratio	=	215.1452			
Akaike Information	=	1.2156	Amemiya Prediction	=	3.3724
DW (Untransformed)	=	1.3089	DW (Transformed)	=	1.9169

$$i_3 = \underset{(-6.24)}{-1.875} + \underset{(36.37)}{0.8787} \; R_{mt}$$

Adjusted R^2 = 0.9720
$F(1,58)$ = 2051.1068

Log-Likelihood Ratio	=	215.6146			
Akaike Information	=	1.1761	Amemiya Prediction	=	3.2421
DW (Untransformed)	=	1.4814	DW (Transformed)	=	1.8756

$$i_4 = \underset{(-6.46)}{-1.845} + \underset{(41.32)}{0.9458}\ R_{mt}$$

Adjusted R^2 = 0.9803
F(1,58) = 2938.9089

Log-Likelihood Ratio	=	236.6936			
Akaike Information	=	0.9678	Amemiya Prediction	=	2.6322
DW (Untransformed)	=	1.4151	DW (Transformed)	=	1.7794

$$i_5 = \underset{(-4.27)}{-1.485} + \underset{(35.63)}{1.0016}\ R_{mt}$$

Adjusted R^2 = 0.9669
F(1,58) = 1728.9607

Log-Likelihood Ratio	=	205.6697			
Akaike Information	=	1.6048	Amemiya Prediction	=	4.9772
DW (Untransformed)	=	1.6730	DW (Transformed)	=	1.9550

$$i_6 = \underset{(1.448)}{0.6374} + \underset{(27.34)}{0.9492}\ R_{mt}$$

Adjusted R^2 = 0.9675
F(1,58) = 1759.555

Log-Likelihood Ratio	=	206.6880			
Akaike Information	=	1.5140	Amemiya Prediction	=	4.5467
DW (Untransformed)	=	1.1442	DW (Transformed)	=	1.8965

APPENDIX 5
GLS Regression Results and Across Equation Constraints

Autocorrelations: $\varepsilon_{i,t} = \rho_i \varepsilon_{i,t-1} + u_{i,t}$ $\forall$ i=1,2,3,4,5,6 and $\forall$ t

Eq 1.	0.19309	Eq 7.	0.41233
Eq 2.	0.44025	Eq 8.	0.27788
Eq 3.	0.34000	Eq 9.	0.35915
Eq 4.	0.32908	Eq 10.	0.15887
Eq 5.	0.30189	Eq 11.	0.32077
Eq 6.	0.24957	Eq 12.	0.40075

```
Estimates for equation: R1
Observations          =  60
Mean of LHS           = -0.3715499       Std.Dev of LHS =  12.05379
StdDev of residuals =  2.193787        Sum of squares = 279.1366
R-squared             =  0.9663147       Adj R-squared  =   0.9657339
Durbin-Watson Stat. =  1.9343341       Autocorrelation=   0.0328330
RHO used for GLS      =  0.1930943
Variable   Coeff    Std. Error   t-ratio   Prob|t|≥x   Mean    Std.Dev.
--------------------------------------------------------------------------
Constant  3.3270   0.3643        9.132     0.00000
WAIR      0.98770  0.2906E-01   33.984     0.00000    -3.7461   11.799
```

```
Estimates for equation: T1
Observations          =  60
Mean of LHS           = -4.835717        Std.Dev of LHS =  10.68466
StdDev of residuals =  1.704914        Sum of squares = 168.5904
R-squared             =  0.9741070       Adj R-squared  =   0.9736606
Durbin-Watson Stat. =  1.7740362       Autocorrelation=   0.1129819
RHO used for GLS      =  0.4402469
Variable   Coeff    Std. Error   t-ratio   Prob|t|≥x   Mean    Std.Dev.
--------------------------------------------------------------------------
Constant -1.5921   0.4030       -3.950     0.00008
WAIR      0.87462  0.3120E-01   28.032     0.00000    -3.7461   11.799
```

Estimates for equation: R2
Observations = 60
Mean of LHS = -0.7606334 Std.Dev of LHS = 11.97243
StdDev of residuals = 1.960243 Sum of squares = 222.8680
R-squared = 0.9727382 Adj R-squared = 0.9722682
Durbin-Watson Stat. = 1.7271624 Autocorrelation= 0.1364188
RHO used for GLS = 0.3399970

Variable	Coeff	Std. Error	t-ratio	Prob|t|≥x	Mean	Std.Dev.
Constant	2.8695	0.3956	7.255	0.00000		
WAIR	0.98083	0.3109E-01	31.547	0.00000	-3.7461	11.799

Estimates for equation: T2
Observations = 60
Mean of LHS = -4.998783 Std.Dev of LHS = 10.75899
StdDev of residuals = 1.697315 Sum of squares = 167.0909
R-squared = 0.9746906 Adj R-squared = 0.9742543
Durbin-Watson Stat. = 1.7696864 Autocorrelation= 0.1151568
RHO used for GLS = 0.3290765

Variable	Coeff	Std. Error	t-ratio	Prob|t|≥x	Mean	Std.Dev.
Constant	-1.7206	0.3372	-5.102	0.00000		
WAIR	0.88412	0.2656E-01	33.289	0.00000	-3.7461	11.799

Estimates for equation: R3
Observations = 60
Mean of LHS = -0.7498834 Std.Dev of LHS = 11.70923
StdDev of residuals = 2.267963 Sum of squares = 298.3320
R-squared = 0.9618482 Adj R-squared = 0.9611904
Durbin-Watson Stat. = 1.8463782 Autocorrelation= 0.0768109
RHO used for GLS = 0.3018863

Variable	Coeff	Std. Error	t-ratio	Prob|t|≥x	Mean	Std.Dev.
Constant	2.7881	0.4333	6.435	0.00000		
WAIR	0.95267	0.3418E-01	27.872	0.00000	-3.7461	11.799

Estimates for equation: T3
Observations = 60
Mean of LHS = -5.157517 Std.Dev of LHS = 10.59054
StdDev of residuals = 1.704722 Sum of squares = 168.5524
R-squared = 0.9736507 Adj R-squared = 0.9731964
Durbin-Watson Stat. = 1.8334655 Autocorrelation= 0.0832673

```
RHO used for GLS     =  0.2495651
Variable   Coeff    Std. Error   t-ratio   Prob|t|≥x   Mean    Std.Dev.
------------------------------------------------------------------------
Constant -1.9046   0.3037       -6.271     0.00000
WAIR      0.87108  0.2410E-01   36.151     0.00000   -3.7461  11.799


Estimates for equation: R4
Observations          =  60
Mean of LHS           = -0.2302333        Std.Dev of LHS =  11.89623
StdDev of residuals =  2.364248          Sum of squares = 324.2008
R-squared             =  0.9598333        Adj R-squared  =   0.9591408
Durbin-Watson Stat. =  1.9077580         Autocorrelation=   0.0461210
RHO used for GLS     =  0.4123331
Variable   Coeff    Std. Error   t-ratio   Prob|t|≥x   Mean    Std.Dev.
------------------------------------------------------------------------
Constant  3.3151   0.5340        6.208     0.00000
WAIR      0.95912  0.4162E-01   23.046     0.00000   -3.7461  11.799


Estimates for equation: T4
Observations          =  60
Mean of LHS           = -5.387633         Std.Dev of LHS =  11.37502
StdDev of residuals =  1.513637          Sum of squares = 132.8837
R-squared             =  0.9819931        Adj R-squared  =   0.9816827
Durbin-Watson Stat. =  1.8078974         Autocorrelation=   0.0960513
RHO used for GLS     =  0.2778803
Variable   Coeff    Std. Error   t-ratio   Prob|t|≥x   Mean    Std.Dev.
------------------------------------------------------------------------
Constant -1.8586   0.2807       -6.622     0.00000
WAIR      0.94229  0.2232E-01   42.221     0.00000   -3.7461  11.799


Estimates for equation: R5
Observations          =  60
Mean of LHS           =  1.747283         Std.Dev of LHS =  14.45791
StdDev of residuals =  3.680209          Sum of squares = 785.5484
R-squared             =  0.9341079        Adj R-squared  =   0.9329719
Durbin-Watson Stat. =  2.0351433         Autocorrelation=  -0.0175717
RHO used for GLS     =  0.3591457
Variable   Coeff    Std. Error   t-ratio   Prob|t|≥x   Mean    Std.Dev.
------------------------------------------------------------------------
Constant  6.0508   0.7678        7.881     0.00000
WAIR      1.1634   0.6075E-01   19.152     0.00000   -3.7461  11.799
```

```
Estimates for equation: T5
Observations          =  60
Mean of LHS           = -5.243667       Std.Dev of LHS =  12.07821
StdDev of residuals =  2.131881         Sum of squares = 263.6051
R-squared             =  0.9683175      Adj R-squared  =   0.9677712
Durbin-Watson Stat. =  1.9643071        Autocorrelation=   0.0178464
RHO used for GLS      =  0.1588726
Variable   Coeff   Std. Error   t-ratio   Prob|t|≥x   Mean   Std.Dev.
-----------------------------------------------------------------------
Constant -1.5094   0.3411       -4.425    0.00001
WAIR      0.99470  0.2742E-01   36.270    0.00000   -3.7461  11.799


Estimates for equation: R6
Observations          =  60
Mean of LHS           =  0.1265334      Std.Dev of LHS =  12.41813
StdDev of residuals =  2.293773         Sum of squares = 305.1608
R-squared             =  0.9653033      Adj R-squared  =   0.9647051
Durbin-Watson Stat. =  1.8962689        Autocorrelation=   0.0518655
RHO used for GLS      =  0.3207744
Variable   Coeff   Std. Error   t-ratio   Prob|t|≥x   Mean   Std.Dev.
-----------------------------------------------------------------------
Constant  3.9114   0.4503        8.687    0.00000
WAIR      1.0118   0.3548E-01   28.518    0.00000   -3.7461  11.799


Estimates for equation: T6
Observations          =  60
Mean of LHS           = -2.862067       Std.Dev of LHS =  11.64252
StdDev of residuals =  1.910137         Sum of squares = 211.6203
R-squared             =  0.9726262      Adj R-squared  =   0.9721543
Durbin-Watson Stat. =  1.7016911        Autocorrelation=   0.1491545
RHO used for GLS      =  0.4007490
Variable   Coeff   Std. Error   t-ratio   Prob|t|≥x   Mean   Std.Dev.
-----------------------------------------------------------------------
Constant  0.64350  0.4235        1.520    0.12862
WAIR      0.95226  0.3307E-01   28.794    0.00000   -3.7461  11.799
```

Hypothesis Tests:

$$r_{1t} = \alpha_{1L} + \beta_{1L}R_{mt} + \varepsilon_{1Lt}$$
$$r_{2t} = \alpha_{2L} + \beta_{2L}R_{mt} + \varepsilon_{2Lt}$$
$$r_{3t} = \alpha_{3L} + \beta_{3L}R_{mt} + \varepsilon_{3Lt}$$
$$r_{4t} = \alpha_{4L} + \beta_{4L}R_{mt} + \varepsilon_{4Lt}$$
$$r_{5t} = \alpha_{5L} + \beta_{5L}R_{mt} + \varepsilon_{5Lt}$$
$$r_{6t} = \alpha_{6L} + \beta_{6L}R_{mt} + \varepsilon_{6Lt}$$

$$i_{1t} = \alpha_{1D} + \beta_{1D}R_{mt} + \varepsilon_{1Dt}$$
$$i_{2t} = \alpha_{2D} + \beta_{2D}R_{mt} + \varepsilon_{2Dt}$$
$$i_{3t} = \alpha_{3D} + \beta_{3D}R_{mt} + \varepsilon_{3Dt}$$
$$i_{4t} = \alpha_{4D} + \beta_{4D}R_{mt} + \varepsilon_{4Dt}$$
$$i_{5t} = \alpha_{5D} + \beta_{5D}R_{mt} + \varepsilon_{5Dt}$$
$$i_{5t} = \alpha_{6D} + \beta_{6D}R_{mt} + \varepsilon_{6Dt}$$

Joint test of restrictions: $\beta_{iI} = \beta_{iL} - \beta_{iD}$ $i=1,2,3,4,5,6$

H_0: $\{\beta_{1I} = \beta_{2I} = \beta_{3I} = \beta_{4I} = \beta_{5I} = \beta_{6I}\} = 0$

H_A: $\{\beta_{1I} = \beta_{2I} = \beta_{3I} = \beta_{4I} = \beta_{5I} = \beta_{6I}\} \neq 0$

Wald Statistic = 169.5237. Prob from $\chi^2[6]$ = 0.00000

Individual test of restriction: Fncn(i)

H_0: $\{\beta_{iL} - \beta_{iD}\} = 0$

H_A: $\{\beta_{iL} - \beta_{iD}\} \neq 0$

Variable	Coeff	Std.Error	t-ratio	Prob|t|≥x	Conclusion
Fncn(1)	0.113080	0.01898	5.957	0.00000	Reject H_0
Fncn(2)	0.096715	0.01932	5.005	0.00000	Reject H_0
Fncn(3)	0.081599	0.02169	3.761	0.00017	Reject H_0
Fncn(4)	0.016830	0.03470	0.485	0.62767	Fail to Reject H_0
Fncn(5)	0.168700	0.06307	2.675	0.00747	Reject H_0
Fncn(6)	0.059495	0.02856	2.083	0.03725	Reject H_0