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Principal and Agent in a Lexicographic Model

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Abstract

This paper formulates the principal-agent problem in a lexicographic arbitration framework. Applying a previous result, the solution satisfies four conditions similar to those of Nash (1950) and is the only solution that does so. Less risk aversion in the sense of this paper implies a riskier choice, and there is a rationale for the agent's fee to be a linear function of the monetary outcome.

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1. Introduction

The principal-agent problem concerns the determination of the agent's fee so that the agent in pursuit of his own interests will also advance those of the principal. It is usually assumed[1/] that the two parties have von Neumann-Morgenstern utility functions which, however, typically lead to complicated formulas for the agent's fee in contrast to the simple linear functions commonly observed (Stiglitz (1987)).

In this paper we will assume lexicographic preferences and view the problem as a particular case of an arbitration model. Under the assumptions, the solution satisfies four properties similar to those of Nash (1950) and it is the only one that does so. It will be seen that in the sense of this paper, less risk aversion implies a riskier choice. There is also a simple explanation for the agent's fee to be a linear function of the monetary outcome.

2. An arbitration framework

Consider two persons who believe that by entering into an agreement they can do better than otherwise. Let x^h be the decision of person $h = 1, 2$, and assume that h evaluates the alternative $x = (x^1, x^2)$ in terms of the vector $u^h(x) = (u_1^h(x), u_2^h(x), \ldots)$ where u_i^h is a continuous function that orders the alternatives on the basis of his ith criterion of choice. Letting $xP_i^h y$ mean that $u_i^h(x) > u_i^h(y)$ and $xI_i^h(y)$ mean $u_i^h(x) = u_i^h(y)$, assume that h prefers x to y

if $xP_j^h y$ for some j and $xI_i^h y$ for all $i < j$. Writing $u_i(x) = (u_i^1(x), u_i^2(x))$ and $u(x) = (u_1(x), u_2(x), \ldots)$, let $U(A) = \{u(x) | x \in A\}$ where A is the set of admissible alternatives. We assume that $U(A)$ is compact and connected. Defining

$$A_i = \{x \in A_{i-1} | \forall y \in A_{i-1}: (\exists h: yP_i^h x) \rightarrow (\exists k: xP_i^k)\}, \quad i = 1, 2, \ldots$$

with $A_0 = A$, put

$$A^* = A \cap A_1 \cap A_2 \cap \ldots$$

as the solution to the problem of finding an x that might be considered fair and mutually acceptable.

A_i is the set of points in A_{i-1} that may be called u_i-optimal (in the Pareto sense), and A_1 would be the usual Pareto-optimal points if each h had only the real valued utility function u_1^h. There would be no obvious way of making a choice in A_1 if an arbiter were to consider only u_1, but $A_2 \subset A_1$ narrows the choice by selecting points that are also u_2-optimal. A further narrowing down makes use of u_3. If u_3^1 and u_3^2 give the same ordering on A_2 and therefore both parties will make the same choice on A_2, then $U(A_3) = U(A^*)$ is a singleton set. In this case, denoting a possible solution by $g(A)$ not necessarily the solution $A^* = g^*(A)$, Lemmas 1 and 2 in Encarnación (1986)[2/] directly give

<u>Proposition 1.</u> $g = g^*$ if and only if g satisfies the following conditions 1 to 4.

Condition 1 (invariance). The solution $g(A)$ is unchanged by an arbitrary positive monotonic transformation u_i^h $(i = 1, 2, 3;\ h = 1, 2)$.

Condition 2 (symmetry). If $U(A)$ is symmetrical, then $U(g(A)) = \{\bar{u}\}$ with $\bar{u}^1 = \bar{u}^2$. ($U(A)$ is symmetrical if for every x, $u(x) \in U(A)$ implies $\exists y$: $u(y) \in U(A)$ such that $u^1(x) = u^2(y)$ and $u^1(y) = u^2(x)$.)

Condition 3 (Pareto optimality). No element of $g(A)$ is Pareto inferior to any element of A. (As usual, x is Pareto inferior to y if one party prefers y to x and the other does not prefer x to y.)

Condition 4 (rational choice). If $A \subset A'$ and $A \cap g(A') \neq \emptyset$, then $A \cap g(A') = g(A)$.

Conditions 3 and 4 are the same as those of Nash. Condition 2 is a multi-dimensional version of Nash's. Condition 1 of Nash involved the positive linear transformation property of von Neumann-Morgenstern utility functions that Nash worked with. In the next section a formulation of the principal-agent relationship makes it a special case of the above model.

3. Principal and agent

Let $z = z(q, \sigma)$ be the monetary outcome or result of the agent's activities[3/] $q = (q_1, \ldots, q_n) \geq 0$ if σ is the true state of nature. Letting G be the cumulative distribution function of z, assume that $\partial(1 - G(z'))/\partial q_r > 0$ $(r = 1, \ldots, n)$ for arbitrary z' except of course where $G(z') = 1$. Write the principal's ordinal utility (of the usual kind) as $W(z - f(z), c)$ where $f(z)$ is the fee paid to the agent, so $z - f(z)$ is the principal's share, and the parameter c is the psychic cost of being dependent on the agent for the outcome. We assume that because of safety-first considerations--see e.g. Telser (1955)--the principal is concerned with the probability $\pi(f, q) = \Pr\{W(z - f(z), c) \geq W^*\}$, W^* being a satisficing level, but he would find it acceptable if $\pi(f, q) \geq \alpha$.

The idea of an acceptable probability α is familiar from the classical Neyman-Pearson rule which considers a specified probability of avoiding a Type I error as acceptable; the standard statistical practice of taking some probability level as good enough for the purpose, say, of detecting batches of items containing more than a certain fraction of defectives is similar. Terms like "reasonable risks" and "acceptable risks" carry the same idea. It is natural then to say that the value of α reflects the principal's degree of risk aversion: he is more risk averse if α is larger. This sense of risk aversion is different from the usual one in the literature--see e.g. Machina and Rothschild (1987)--but conforms to the dictionary meaning of risk as chance of loss. Loss here would occur with an outcome $z < z^*$, where z^* is given by $W(z^* - f(z^*), c) = W^*$.

Similarly, assume that the agent's ordinal utility is $V(f(z), q)$ with $\partial V/\partial q_r < 0$ all r and satisficing level V^*, and that he would consider it acceptable if $\phi(f, q) = \Pr\{V(f(z), q) \geq V^*\}$ is not less than β. We will refer to $\phi(f, q) \geq \beta$ as the agent's safety condition.

Suppose that in order of priority the principal's criteria of choice are $\min\{\pi(f, q), \alpha\}$, the expected value $E(z - f(z))$, and $E(z)$.

Since $f(z) = a_0 + a_1 z + \ldots + a_m z^m$ can be characterized by the coefficients $a_0, a_1, \ldots, a_m$, we can represent f as a vector and write $(f, q) = (x^1, x^2) = x$ using the notation of Section 2. Put

$$u_1^1(x) = g_1^1(\min\{\pi(x), \alpha\})$$

$$u_2^1(x) = g_2^1(E\{z - f(z)\}) = g_2^1(H(x))$$

$$u_3^1(x) = g_3^1(E(z)) = g_3^1(K(x))$$

where each g_i^1 is an arbitrary positive monotonic function and therefore so is each u_i^1. Similarly for the agent,

$$u_1^2(x) = g_1^2(\min\{\phi(x), \beta\})$$

$$u_2^2(x) = g_2^2(E\{f(z)\}) = g_2^2(J(x))$$

$$u_3^2(x) = g_3^2(E(z)) = g_3^2(K(x)).$$

Since the third criterion is the same for both parties, Proposition 1 applies, giving the solution say $(f^*, q^*) \in A^*$.

The assumption that both parties have $E(z)$ as a common concern, after their individualistic objectives, formalizes the idea that a person is an agent of another if in some sense he acts in the interest of the principal and therefore internalizes some objective of the latter. Otherwise, if the agent were to be interested only in his fee, it seems doubtful that the principal would want a relationship with him. We assume that the principal will not enter into one unless $\pi(x) \geq \alpha$, and neither will the agent unless $\phi(x) \geq \beta$. To avoid excessive repetition it will be understood in all that follows, unless indicated otherwise, that this requirement is met, i.e. all (f, q) to be considered are such that $(f, q) \in A_1$. The relationship is contracted for one "period", but it could be renewed and maintained if both parties find it satisfactory.

4. The individual maximization problems

Suppose $0 < \alpha < 1$ and $0 < \beta < 1$. Given f, the agent's decision $q = Q(f)$ will maximize $E(f(z)) = J(f, q)$ subject to $\pi(f, q) \geq \alpha$ and $\phi(f, q) \geq \beta$. Write the latter as

$$(1) \qquad C(f, q) \leq B$$

where $C(f, q) = 1 - \phi(f, q)$ and $B = 1 - \beta$. In the neighborhood of the solution to the agent's decision problem, a higher q_r lowers ϕ and increases C, so (1) is like a cost constraint--"cost" being the probability of failing to make the agent's V^* level. The agent will maximize

$$J(f, q) + \lambda(\pi(f, q) - \alpha) + \mu(B - C(f, q))$$

which requires

$$(2) \qquad J_r + \lambda\pi_r - \mu C_r \leq 0 \qquad r = 1, \ldots, n$$

$$(3) \qquad (J_r + \lambda\pi_r - \mu C_r)\, q_r = 0 \qquad r = 1, \ldots, n$$

where $J_r = \partial J/\partial q_r$ (similarly for μ_r and C_r) and the Lagrange multipliers $(\lambda, \mu) \geq 0$. One must have $\mu > 0$ in (2) since $\lambda \geq 0$ and $(J_r, \pi_r, C_r) > 0$. If $\lambda = 0$, the problem reduces to the uninteresting case where $J(f, q)$ is maximized subject only to (1), and the principal's safety condition $\pi(f, Q(f)) \geq \alpha$ plays no essential role in the model of Section 3.[4/] We therefore assume $\lambda > 0$ so that both safety conditions are satisfied as equalities. From (2) and (3),

(4) $J_r/C_r + \lambda\pi_r/C_r = \mu$ if $q_r > 0$

(5) $q_r = 0$ if $J_r/C_r + \lambda\pi_r/C_r < \mu$.

Thus the optimal q will include an activity with a low π_r/C_r if its J_r/C_r is high enough, as well as another activity s with a low J_s/C_s if its π_s/C_s is high enough. We propose to say that r has a higher return than s if $J_r/C_r > J_s/C_s$, and r is more risky (less safe) than s if $\pi_r/C_r < \pi_s/C_s$. (The optimal q may therefore be a mix of high risk high return and low risk low return activities.) With r having a lower π_r/C_r, it contributes less (relative to its marginal cost) towards meeting the π constraint, so it is less safe and more risky for the purpose of making the principal's V^* level. At the same time, with its higher C_r/π_r, it adds more (relative to its marginal contribution to π) to the probability of failing to attain the agent's V^*, and is therefore also risky in this sense.

To show the effects of a lower β (higher B) on the activity-mix, consider any $q_r > 0$, $q_s > 0$, with r riskier, so $J_r/J_s > C_r/C_s > \pi_r/\pi_s$. A higher q_s level made possible by $dB > 0$ is clearly not optimal in the presence of the higher return r. Neither is a simple increase in q_r with no change in q_s, which gives only $J_r/C_r < \mu = \partial J/\partial B$. Instead, suppose $dq_s = -a\, dq_r < 0$ where $a = \pi_r/\pi_s$, which just maintains the π constraint with $d\pi = \pi_r\, dq_r + \pi_s\, dq_s = 0$. Since

$$J_r - a\, J_s = \mu(C_r - a\, C_s)$$

from (4), while $dJ = J_r\, dq_r + J_s\, dq_s = (J_r - a\, J_s)dq_r$ and $dC = (C_r - a\, C_s)dq_r = dB$, one would have $dJ = \mu dB$ as called for by $\partial J/\partial B = \mu$. In other words, there will be a substitution of riskier for safer activities. In short,

<u>Proposition 2</u>. Less risk aversion on the part of the agent implies a riskier hence higher return activity-mix.[5/]

Because of the basic symmetry, it is easy to show a similar result for the principal with f nonlinear in general. Given $q = Q(f)$, his problem is to choose $a_0, a_1, \ldots a_m$ in $f(z) = a_0 + a_1 z + \ldots + a_m z^m$, a_v possibly negative, so as to maximize $E(z - f(z)) = H(f, Q(f))$ subject to $\phi(f, Q(f)) \geq \beta$ and $C^*(f, Q(f)) \leq B^*$ where $C^*(f, Q(f)) = 1 - \pi(f, Q(f))$ and $B^* = 1 - \alpha$. Maximizing $H(.) + \lambda^*(\phi(.) - \beta) + \mu^*(B^* - C^*(.))$ requires

$$(4') \qquad H_v/C_v^* + \lambda^*\, \phi_v/C_v^* = \mu^* \qquad v = 0, 1, \ldots, m.$$

Looking at (4) and (4'), a straightforward adaptation of the previous discussion gives

Proposition 3. Less risk aversion on the part of the principal implies a riskier and higher return mix of coefficients.

5. A positive model

If the agent alone were to decide the matter, he would select that f such that $(f, Q(f)) \in A_1$ maximizes $E(f(z))$, choosing $q = Q(f)$ as described in Section 4; if the principal, the maximand would be $E(z - f(z))$ and he would choose f as described also in Section 4. In general these two decisions would not coincide, making necessary the presence of an arbiter to determine $(f^*, Q(f^*)) \in A^*$ in the model of Section 3. That model seems better thought of as primarily normative since it relies on an arbiter that in fact does not exist. For a positive theory, we now postulate that model as holding without any need for an arbiter.

6. Linearity of the fee function

Let us say that f is admissible if $(f, Q(f))$ is admissible. Suppose that to be admissible, f must be linear: $f(z) = a_0 + a_1 z$. With a_0 and a_1 providing only two degrees of freedom, it is clear that $\pi(f, Q(f)) = \alpha$ and $(f, Q(f)) = \beta$ will determine f uniquely, i.e. $\{f \mid (f, Q(f)) \in A_1\}$ is a singleton, and therefore $A_1 = A^*$. This means that once principal and agent have found an f that meets their safety conditions, they need look no farther: they have found $(f^*, Q(f^*)) \in A^*$.

If nonlinear f's are admissible, an arbiter would be required to narrow down the choice from A_1 to A_2 to $A_3 = A^*$. Hence

Proposition 4. $A_1 = A^*$ [i.e., no arbiter is needed to determine $(f^*, Q(f^*)) \in A^*$] if and only if admissible f's are linear.

The positive model of Section 5 therefore implies linear fee functions as commonly observed.

7. Other considerations

In the foregoing we have implicitly assumed that the principal's and agent's utility functions are known to each other. However, in practice the principal's knowledge of the functional relationship $q = Q(f)$ is imperfect.

If $f(z) = a_0 + a_1 z$ with $0 < a_1 < 1$, the agent's decision $q = Q(f)$ will maximize his $E(f(z)) = a_0 + a_1 E(z)$ hence $E(z)$ and therefore also the principal's $E(z - f(z)) = -a_0 + (1 - a_1) E(z)$. If f is not linear, clearly the agent's maximization of $E(f(z)) = E(z) - E(z - f(z))$ will not imply mazimization of the principal's (expected) share. This gives

Proposition 5. The agent's decision implies maximization of the principal's $E(z - f(z))$ if and only if $f(z) = a_0 + a_1 z$ $(0 < a_1 < 1)$.[6/]

Thus if the principal wants to make sure that his interests will be advanced with the agent's when the latter maximizes his fee, f will have to be linear. This provides, at a more practical level, a rationale for the observed facts.

With f linear, the principal would want a smaller value of the riskier coefficient and a higher value of the safer one if his α is higher. In the limiting case $\alpha = 1$ which calls for $W(z - f(z), c) \geq W^*$, the principal in effect requires a sure return of $-a_0 > 0$ given by $W(-a_0, c) = W^*$. That is, $a_1 = 1$ in $f(z) = a_0 + z$.7/ We could therefore say that a_0 is riskier than a_1: a larger a_0 smaller a_1 mix is riskier for the principal.

In the other limiting case $\beta = 1$, the agent requires $f(z) = a_0 > 0$ (so $a_1 = 0$, the principal bearing all the risk8/) where a_0 is given by $V(a_0, q) = V^*$. With $E(f(z)) = a_0$, he maximizes $E(z)$ hence $E(z) - a_0 = E(z - f(z))$ for the principal. Professionals who charge flat fees, e.g. most physicians, would be in this category. In the standard literature, the agent will exert no effort in a flat fee arrangement. This might be plausible in a single-shot relationship (and only if considerations of reputation are ignored) but surely not in one where the agent might be interested in maintaining the relationship.

In practice, each party makes his decisions only on the basis of beliefs which may turn out to be mistaken. If it turns out that $z < z^*$ "too often"--recall z^* in $W(z^* - f(z^*), c) = W^*$--the principal would have reason to think that his original belief about $\pi(f,q) \geq \alpha$ was false. He might then conclude that the agent does not exert enough effort, is careless, unreliable or incompetent. On the other hand, if the principal finds that the outcomes over time have been satisfactory, he may take the psychic cost parameter c to be lower than it was initially, permitting a lower value of z^*. This may explain why the principal can tolerate poorer results from an "old" agent whose performance had been adequate in the past, but not the same results from a new agent. The latter is not likely to be retained.

In similar fashion, the agent would feel underpaid if $V(f(z), q) < V^*$ too often through no fault of his own. He may then want to discontinue the relationship.

8. Concluding remarks

A previous paper has shown that if the two parties in a bargaining problem have lexicographic preferences, there is an arbitration scheme which is simply a repeated application of the Pareto optimality principle. If the two parties have the same third objective, the solution satisfies four Nash-type conditions and is the only solution that does so. In the present paper we have placed the principal-agent relationship in that framework. Pursuing safety first, the principal will not put at excessive risk the attainment of a satisficing utility level. His second objective is to maximize his share of the monetary outcome. Symmetrically, the agent has similar criteria, and their common objective is to maximize the value of the outcome.

Viewing risk as chance of loss, its dictionary meaning, loss is incurred when utility is less than its satisficing level. The degree of risk aversion is then the acceptable probability of no loss. As one should require, less risk aversion on the part of the agent leads him to make a riskier choice.

Finally, since in fact there is no arbiter, the positive model of this paper implies that the agent's fee must be a linear function of the outcome. An additional rationale for this common observation derives from the practical consideration that with a linear function, the agent's decision maximizing his fee must also maximize the principal's share, and it is the only functional form that does so.

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Notes

1. See e.g. Spence and Zeckhauser (1971), Ross (1973), Mirrlees (1976), Holmström (1979), Shavell (1979), Grossman and Hart (1983), Radner (1985), Page (1987).

2. In this reference the components of x are quantities of goods, but x is clearly capable of a more general interpretation.

3. Alternatively, instead of productive activities in an obvious sense, the components of q might be thought of as pertaining to different dimensions of work quality like carefulness and reliability in addition to the usual effort level.

4. If $\lambda = 0$, the principal's safety condition would be a nonbinding constraint also in his problem of maximizing $E(z - f(z)) = H(f, Q(f))$ by choice of $(f, Q(f)) \in A_1$.

5. It is worth noting that the usual expected utility framework, with its meanings for risk and risk aversion different from those in this paper, has not yielded a similar proposition.

6. The major problem in standard theory of finding an appropriate balance between risk sharing and incentives is thus bypassed: if the two parties enter into a relationship, they find the risk sharing acceptable because their safety conditions are satisfied, and since the fee function

is linear, the incentives are there.

7. One says in this case that the agent takes all the risk, but his safety condition would still be satisfied, generally as a strict inequality.

8. A similar remark as in footnote 7 applies here also.

References

Encarnación, J. (1986), "Lexicographic Arbitration," Economics Letters, 21, 231-234.

Grossman, S.J. and Hart, O.D. (1983), "An Analysis of the Principal-Agent Problem," Econometrica, 51, 7-45.

Holmström, B. (1979), "Moral Hazard and Observability," Bell Journal of Economics, 10, 74-91.

Machina,, M.J. and Rothschild, M. (1987), "Risk," in The New Palgrave: A Dictionary of Economics, (ed. J. Eatwell, M. Milgate and P. Newman), London: Macmillan.

Mirrlees, J.A. (1976), "The Optimal Structure of Incentives and Authority within an Organization," Bell Journal of Economics, 7, 105-131.

Nash, J.F. (1950), "The Bargaining Problem," Econometrica, 18, 155-162.

Page, F.H. (1987), "The Existence of Optimal Contracts in the Principal-Agent Model," Journal of Mathematical Economics, 16, 157-167.

Radner, R. (1985), "Repeated Principal-Agent Games with Discounting," Econometrica, 53, 1173-1198.

Ross, S.A. (1973), "The Economic Theory of Agency: The Principal's Problem," American Economic Review, 63(2), 134-139.

Shavell, S. (1979), "Risk Sharing and Incentives in the Principal and Agent Relationship," Bell Journal of Economics, 10, 55-73.

Spence M. and Zeckhauser, R. (1971), "Insurance Information, and Individual Action," American Economic Review, 61(2), 380-387.

Stiglitz, J.E. (1987), "Principal and Agent," in The New Palgrave: A Dictionary of Economics (ed. J. Eatwell, M. Milgate and P. Newman), London: Macmillan.

Telser, L.G. (1955). "Safety First and Hedging," Review of Economic Studies, 23, 1-16.