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Arbitration and Group Decision under Uncertainty*

by

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Abstract

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Assuming that the parties to a conflict have lexicographic preferences, an arbitration model is formulated whose solution is fair in the sense that it is not arbitrary. The solution, which is simply an extension of the idea of Pareto optimality to the multidimensional utility case, satisfies four conditions analogous to those of Nash for a bargaining problem, and it is the only solution that does so. It also applies to the group decision problem under uncertainty, permitting different individual preference orderings and different subjective probability judgements.

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Suppose a set of possible alternatives that require joint action by two individuals or parties. A conflict arises when, if either person were to have his way, he would prefer an alternative that is different from what the other person would choose. If the parties to the conflict feel that some agreement is better than none, an arbiter is then needed to decide for both of them in a reasonable manner which, inter alia, does not unfairly favor one over the other. The set of possible alternatives can be narrowed to what may be called the admissible set if the arbiter applies from the outset the norms or standards generally accepted in the society--including any on equity--to eliminate alternatives that violate those standards. A problem remains, however, when there are still many Pareto-optimal elements even in the smaller admissible set.

In the context of game theory, Luce and Raiffa (1957, p. 121) "define an arbitration scheme to be a function ... which associates to each conflict ... a unique payoff to the players." For example, the Nash (1950) solution maximizes the product of the two players' von Neumann-Morgenstern utilities, putting status quo at the origin. What is especially interesting about the Nash solution is that given the basic assumption about the utility functions, it satisfies four conditions which appear reasonable in the context, and it is the only solution that does so. Luce and Raiffa (1957, pp. 349-350) have extended the Nash solution to the n-person case.

In this paper we will consider the problem under the assumption that preferences are representable by lexicographic utility functions; see Fishburn (1974) for a survey of this literature. Our solution, which is simply a natural extension of the idea of Pareto optimality to the multidimensional utility case, satisfies four properties analogous to the Nash conditions and is the only solution that does so. After a brief review of lexicographic preferences in Section I, the solution and some of its properties are described in Sections II and III. Sections IV and V extend the discussion to group decision and uncertainty, and Section VI concludes the paper.

I. Lexicographic Utility

It is assumed that one evaluates alternatives in terms of multiple criteria corresponding to essentially different wants and objectives, so that the utility of an alternative x is a vector $u(x) = (u_1(x), u_2(x), \ldots)$. The real valued function u_i, similar to the standard utility function in admitting of arbitrary positive monotonic transformations, ranks the x's on the basis of the ith criterion of choice, which is more important and has higher priority than the jth if $i < j$. Given a particular u_i, it is postulated that there is a number u_i^* such that if $u_i(x) \geq u_i^*$, x is considered satisfactory or acceptable with respect to the ith criterion. Writing $v_i(x) = \min\{u_i(x), u_i^*\}$ and $v(x) = (v_1(x), v_2(x), \ldots)$, we then say that x is preferred to y if and only if the first nonvanishing component of $v(x) - v(y)$ is positive, i.e. the preference ordering of the x's corresponds to the lexicographic ordering of the $v(x)$'s. We might call $v(x)$ the virtual utility of x since in the determination of choice it is what counts and not $u(x)$.

Let us call such a preference system an L^*-ordering in contrast to L-ordering where $u_i^* = \infty$ for all i.[1/] Many economists are correctly skeptical about the analytical usefulness of lexicographic utility functions viewing the latter in terms of L-ordering. With L-ordering, unless it happens that the set of x's that maximize u_1 is not a singleton (one-element set), the first component of the utility vector will suffice to determine choice in which case the other components are superfluous. With L^*-ordering on the other hand, a less important criterion plays a role in choice once the more important criteria have been satisfied. More precisely, let

$$S_i = \{x \in S_{i-1} \mid v_i(x) = \max_y \{v_i(y) \mid y \in S_{i-1}\}\}$$

$i = 1, 2, \ldots,$ where S_0 is the set of possibilities. We have $S_i \subset S_{i-1}$, and $S_i \neq S_{i-1}$ except where S_{i-1} is already a singleton or where v_i is vacuous in the sense that it does not narrow the choices in a many-element set. If S_j is a singleton and S_{j-1} is not, one's decision then maximizes $u_j(x)$ subject to $x \in S_i$ $(i = 0, 1, \ldots, j-1)$. We note that if S_i is a singleton whenever $\max_y \{v_i(y) \mid y \in S_{i-1}\} < u_i^*$, the fact that u_j is the maximand means that for some $x \in S_0$, one has $u_i(x) \geqq u_i^*$ for all $i < j$; in other words, all the criteria more important than u_j are satisfied. In general, the larger is the initial field of choice S_0 (the budget constraint set in consumer theory, for example), the higher would be the index j of the maximand.

It should be noted that nothing prevents the possibility of some criterion $u_r = f(u_1, \ldots, u_{r-1})$ so that u_r is a composite criterion so to speak. If (a) $\exists\, x \in S_0 : u_i(x) > u_i^*$ for all $i < r$ and (b) S_r is a singleton, a person's decision problem would simply be to maximize $u_r(x)$ subject only to $x \in S_0$ since $u_i(x) \geq u_i^*$ $(i < r)$ are not binding constraints. The usual real valued utility function can thus be thought of as the special case where S_0 is such that (a) holds so that substitution possibilities exist among the arguments of f and therefore also among the goods on which these arguments depend. But if S_0 is sufficiently limited, some u_i with $i < r$ would be the maximand and those substitution possibilities would also be limited. For example, the need for food is more important than the need for leisure, and a man paid a bare subsistence wage will not reduce his food intake in order to have more leisure. Nor would Plato's "just" man sacrifice an iota of justice for any amount of wealth.

Chipman (1960) has made clear, after the lead of Georgescu-Roegen (1954), that the validity of real valued utility functions is based on the Principle of Substitution, which says that one would be willing to give up some amount of anything for at most a finite amount of something else. The principle is of course false as a general proposition, and it is not necessary for an analytical representation of preferences and choice. While it is a convenient simplication that is useful in many special cases, it cannot be invoked as an argument for real valued utility functions without circularity: the two are essentially equivalent.

II. The Solution

Consider two persons indexed by $h = 1, 2$ who agree to arbitration over their conflict because they would do better through an arbitrated solution than if they were to go their separate ways. They have L*-orderings but their criteria of choice can be different. Using superscripts on the notation in Section I to label the persons, h looks at the alternatives in terms of his $v^h = (v_1^h, v_2^h, \ldots)$. We now write $u_i(x) = (u_i^1(x), u_i^2(x))$ in $u(x) = (u_1(x), u_2(x), \ldots)$ and similarly $v_i(x) = (v_i^1(x), v_i^2(x)$ in $v(x) = (v_1(x), v_2(x), \ldots)$. If S is a set of alternatives, we can thus represent $x \in S$ as a point $u(x) \in U(S) = \{u(x) | x \in S\}$ or as a point $v(x) \in V(S) = \{v(x) | x \in S\}$. We will also write $U_i(S) = \{u_i(x) | x \in S\}$ and $V_i(S) = \{v_i(x) | x \in S\}$.

Since $u(x)$ is unique for each x, $u_i(x)$ "determines" $u_{i+1}(x)$ in $U(S)$; we may thus say that $u_{i+1} = F_i(u_i)$, $i = 1, 2, \ldots$. As usual, S is finite or has the cardinality of the continuum. In the latter case, it is natural to assume that F_i is continuous. Hence if $U_i(S)$ is a compact set, $U_{i+1}(S)$ is also compact.

Denoting the nonempty admissible set by A, we assume that $U_1(A)$ is compact. Let

$$A_i = \{x \in A_{i-1} | \forall y \in A_{i-1}: \ (\exists h: v_i^h(y) > v_i^h(x) \rightarrow \exists k : v_i^k(x) > v_i^k(y))\}, \qquad i = 1, 2, \ldots$$

where $A_0 = A$. A_1 is the set of Pareto v_1-optimal elements in A. (We will simply say that the elements of A_i are v_i-optimal.) A_1 is nonempty since $U_1(A)$, hence $V_1(A)$, is closed. If $x \in A_1 \subset A$, there is no y in A that every h considers at least as good as x in terms of his v_1^h and someone considers better. Confining himself to

$v_1 = (v_1^1, v_1^2)$ only, the arbiter would have no way of deciding the matter if $V_1(A)$ is not a singleton. (This of course is the usual situation with standard utility functions.) By turning to v_2 however, the arbiter's possible decisions can be narrowed to $A_2 \subset A_1$. A_2, the set of v_2- as well as v_1-optimal points,[2/] is nonempty (because $V_2(A_1)$ is closed since $U_1(A_1)$, hence $U_2(A_1)$, is compact) and smaller than A_1 unless both v_2^1 and v_2^2 happen to be vacuous with respect to A_1. Proceeding further, we put

$$A^* = A \cap A_1 \cap A_2 \cap \cdots$$

as the solution to our arbitration problem. The solution is fair in the sense that it is not arbitrary, for it merely extends the idea of Pareto optimality to a multidimensional framework.

What is interesting is that the more objectives or criteria of choice that the parties to the conflict might have, the easier it is for the arbiter to arrive at a solution: every additional v_i narrows down his choices to $A_i \subset A_{i-1}$. We would thus expect that with enough criteria, A^* will be a singleton. This is obvious where A is a finite set. If it is not, we have the following proposition.

<u>Lemma</u> 1. If v^h is infinite dimensional $(h = 1, 2)$, $V(A^*)$ is a singleton.

<u>Proof:</u> Define a measure $s_i = s(V(A_i))$ of the "size" of $V(A_i)$ by

$$s_i = \sum_{j=1}^{\infty} 2^{-j} \max_{(x,y)} \{d_j(x,y) | \; x,y \in A_i\}$$

where

$$d_j(x,y) = \left| \frac{v_j(x)}{|v_j(x)|} - \frac{v_j(y)}{|v_j(y)|} \right|$$

is a measure of distance between $v_j(x)$ and $v_j(y)$, and $|w| = (w \cdot w)^{1/2}$. Since $s_{i-1} \geq s_i \geq 0$ for all i, the sequence $\{s_i\}$ is convergent. We can rule out the possibility of some j such that $s_i = s_{i-1} > 0$ for all $i > j$, for this would mean an infinite number of pairs v_i^1, v_i^2 which are both vacuous after j. (The coincidence would have probability zero, in addition to making v^h essentially finite.) Therefore $s_i < s_{i-1}$ for an infinite number of indices i, and clearly there is some $\lambda < 1$ such that $s_i \leq \lambda s_{i-1}$ for those i's since A_i excludes most of the elements of A_{i-1}, viz. those which are not v_i-optimal. Thus $\{s_i\}$ converges to 0 and the sequence $\{V(A_i)\}$ converges to a point.

In our view, an infinite dimensional v^h is a straightforward interpretation of the common statement that wants are unlimited. This statement is usually interpreted to mean an unbounded real valued utility function, but the more literal meaning is that the satisfaction of any number of wants implies another want. Alternatively, we can interpret the first t_1 components of v^h as criteria of choice pertaining to the consequences resulting in time period 1, the next t_2 components as criteria pertaining to time period 2, and so on, which incidentally would dispense with the need for time discounting in the case of infinite time horizons (Encarnación 1983a).

What may appear objectionable about the solution A^* is that we are letting lower priority criteria decide, in effect, the choice as regards a higher priority one. But this seems precisely what is done when one does not know how the alternatives rank in terms of the more important criterion. For example, in deciding where to have a good dinner in a strange city, one might not know how the possibilities rank in terms of quality and therefore cannot decide on the basis of this criterion. However, if one has a lower priority preference for Japanese food, the field of choice can be narrowed by considering only Japanese restaurants. The arbiter is in a similar position for he has no way of selecting a point in A_{i-1} without being arbitrary, so he lets A_i narrow down the choice.

II. Properties of the Solution

We will say that $V(S)$ is symmetrical if for every x, $v(x) \in V(S)$ implies $\exists\, y: v(y) \in V(S)$ where $v^1(x) = v^2(y)$ and $v^1(y) = v^2(x)$; similarly, $V_i(S)$ is symmetrical if for every x, $v_i(x) \in V_i(S)$ implies $\exists\, y: v_i(y) \in V_i(S)$ where $v_i^1(x) = v_i^2(y)$ and $v_i^1(y) = v_i^2(x)$.

Denoting a possible solution of A by $g(A)$, not necessarily the solution $A^* = g^*(A)$, consider the following properties that one might require of g.

Condition 1 (invariance): The solution $g(A)$ is unchanged by arbitrary positive monotonic transformations of u_i^h $(i = 1, 2, \ldots;$ $h = 1, 2)$.

Condition 2 (symmetry): If $V(A)$ is symmetrical, then $V(g(A)) = \{\bar{v}\}$, say, where $\bar{v}^1 = \bar{v}^2$.

Condition 3 (Pareto optimality): No element of $g(A)$ is Pareto inferior to any element of A. (As usual, x is Pareto inferior to y if someone prefers y to x and no one prefers x to y.)

Condition 4 (rational choice): If $A \subset A'$ and $A \cap g(A') \neq \emptyset$, then $A \cap g(A') = g(A)$.

These conditions are analogous to those that characterize the Nash (1950) solution to a bargaining problem (cf. Luce and Raiffa 1957, pp. 126-127), allowing for the differences in basic assumptions. Regarding Condition 1, Nash instead assumes von Neumann-Morgenstern utility functions which are unique up to positive linear transformations. Our Condition 2 calls for symmetry in every dimension, while that of Nash is unidimensional. Conditions 3 and 4 are the same as Nash, Condition 4 being Arrow's requirement for a "rational" choice function (Arrow 1959, Definition C4).

The Nash solution is a singleton and has the interesting property that it satisfies his four conditions and is the only solution that does so. In order to get comparable results we will assume that $V(A^*)$ is a singleton; otherwise, Condition 2 would be unreasonable.

Lemma 2. If $g = g^*$, g satisfies Conditions 1 to 4.

Proof. Since a positive monotonic transformation of u_i^h carries along u_i^{h*}, Condition 1 is obviously satisfied.

Assume the hypothesis of Condition 2. Since $V_1(A)$ is symmetrical, so is $V_1(A_1)$ which is just the northeast boundary of $V_1(A)$. Therefore, from the fact that $V(A)$ is symmetrical, so is $V(A_1)$. Thus $V_2(A_1)$ is symmetrical, whence also $V_2(A_2)$, and therefore $V(A_2)$. Repeating the argument, $V(A_i)$ is symmetrical for all i, so the conclusion of Condition 2 follows.

Condition 3 is clear since x is Pareto inferior to some y in A

only if x does not belong to A_i for some i, which would contradict $x \varepsilon A^*$.

To establish Condition 4, let its hypothesis hold. With $V(A^*)$ a singleton, we need only show that (i) $A \cap g^*(A') \subset g^*(A)$, which is false only if there is a z such that (ii) $z \varepsilon A \cap A_1' \cap A_2' \cap \ldots$ but (iii) $z \varepsilon A - A^*$. Suppose such a z. From $z \varepsilon A \cap A_1'$ in (ii) and the fact that $A \subset A'$, we directly have $z \varepsilon A_1$.[3/] Thus $z \varepsilon A_1 \cap A_2'$ using (ii). Since z belongs to A_1 and is v_2-optimal in the larger set A', clearly it is v_2-optimal in A_1, i.e. $A_1 \cap A_2' \subset A_2$, and therefore $z \varepsilon A_2$. Repetition of the argument gives $z \varepsilon A_i$ for all i, which contradicts (iii) and proves (i).

__Lemma__ 3. If g satisfies Conditions 1 to 4, $g = g^*$.

__Proof.__ Using Condition 1 we can put $V(A^*) = \{\hat{v}\}$ where $\hat{v}^1 = \hat{v}^2$ without changing $g(A)$, and we need to show that $V(g(A)) = \{\hat{v}\}$. Let us say that A' symmetrically contains A if for every x, $v(x) \varepsilon V(A') - V(A)$ implies $\exists y: v(y) \varepsilon V(A)$ where $v^1(x) = v^2(y)$ and $v^1(y) = v^2(x)$. Choose A' so that A' symmetrically contains A and V(A') is symmetrical. Then by Condition 2, $V(g(A')) = \{\bar{v}\}$ with $\bar{v}^1 = \bar{v}^2$. Noting that V(A) and V(A') have exactly the same elements v of the form $v^1 = v^2$, the hypothesis of Condition 4 is satisfied, whence $V(g(A)) = \{\bar{v}\}$. Since $\hat{v}_1$ is v_1-optimal, $\bar{v}_1 > \hat{v}_1$ is not possible, and $\bar{v}_1 < \hat{v}_1$ means that g(A) violates Condition 3; hence, $\bar{v}_1 = \hat{v}_1$. With this in hand, the argument can be repeated with respect to v_2 to get $\bar{v}_2 = \hat{v}_2$, etc., so that $\bar{v} = \hat{v}$ as required.

From Lemmas 2 and 3 we therefore have a characterization of $A^* = g^*(A)$ on the assumption that $V(A^*)$ is a singleton.

<u>Theorem.</u> $g = g^*$ if and only if g satisfies Conditions 1 to 4.

IV. Group Decision

It is straightforward to extend the model to the case of n persons. A review of Section II, where the arbiter plays the role of the group decision function, will show that the determination of A^* is not dependent in any way on there being only two persons. A review of Section III will also show that it is applicable to n persons after suitable redefinitions. Let $\pi(e)$ denote a permutation of the components of $e = (e^1, \ldots, e^n)$. We now say that $V(S)$ is symmetrical if for every x, $e = v(x) \in V(S)$ implies that for all $\pi(e)$, $\exists y$: $v(y) \in V(S)$ where $v(y) = \pi(e)$; similarly for $V_i(S)$; and A' symmetrically contains A if for every x, $e = v(x) \in V(A') - V(A)$ implies $\exists(y, \pi(e))$: $v(y) \in V(A)$ where $v(y) = \pi(e)$. The proofs in Section III then apply to the n-person case almost word for word.

Defining the relation R by xRy if $x \in g^*(\{x,y\})$, R is a (total) ordering[4/] since g^* satisfies Condition 4 (Arrow 1959, Theorem 3). Let xR^hy if h prefers x to y or considers them indifferent, and denote his ordering R^h on S by R^h_S. From Murakami's (1961) formulation of Arrow's (1963) celebrated impossibility theorem, the following conditions cannot all be satisfied.

(a) <u>Free triple</u>: There is a set T of three alternatives on which h may have any logically possible R^h_T $(h = 1, \ldots, n)$.

(b) <u>Nondictatorship</u>: There is no h such that for all $x, y \in T$, xP^hy implies xPy, where xP^hy means not yR^hx and xPy means not yRx.

(c) <u>Unanimity</u>: For all x, y, xPy if $xP^h y$ for all h.

(d) <u>Independence of irrelevant alternatives</u> (IIA): $g^*(A)$ is invariant with respect to any changes in $(R^1, \ldots, R^n)$ that leave $(R^1_A, \ldots, R^n_A)$ unchanged.

It is easy to show the compatibility of (a), (b) and (c) with our model, so IIA is perforce violated. Consider $A = \{x, y\}$ where $v^1_1(x) > v^1_1(y)$ and $v^2_1(x) < v^2_1(y)$ so that xP^1y and yP^2x. Since x and y are both v_1-optimal in this 2-person case, $A^* = \{x\}$ if $v_2(x) > v_2(y)$. Suppose now that each h lowers his u^{h*}_2 with the result that $v^h_2(x) = v^h_2(y)$, which leaves R^1_A and R^2_A unchanged. But now, x and y are also v_2-optimal, so $A^* = \{y\}$ if $v_3(y) > v_3(x)$, violating IIA.

The motivation for IIA is simply to make the group decision independent of alternatives outside A (Arrow 1963, p. 26), but it goes farther by making the group decision a function <u>solely</u> of individual preference orderings on A. It thus rules out any group decision function like g^* where the group decision depends on the parameters (the u^{h*}_i in the case of g^*) that determine those orderings. The latter can remain the same even though the parameters change, as in the present example with only two persons and two alternatives--there are no "irrelevant" alternatives--thus violating IIA when $g^*(A)$ changes. We would conclude from this that IIA is overly restrictive, much more so than is usually realized, and that the label "independence of irrelevant alternatives" is apt to mislead. Certainly, no dependence on irrelevant alternatives is needed for a violation.

V. Uncertainty

If the consequences of an alternative are contingent on the state of nature σ, write $u_i^h(x,\sigma)$ instead of $u_i^h(x)$. Let

$$p_i^h(x) = \Pr\{u_i^h(x,\sigma) \geq u_i^{h*}\}$$

and assume a probability level p_i^{h*} such that if $p_i^h(x) \geq p_i^{h*}$, then h considers x satisfactory or acceptable with respect to his ith criterion of choice under uncertainty. The greater is p_i^{h*}, the less is he willing to risk falling short of u_i^{h*}.

The idea that some probabilities are acceptable is clear from the common reference to "acceptable risks" in ordinary language. It is also an old one in the literature. In the classical Neyman-Pearson rule, a specified probability of avoiding a Type I error is taken as satisfactory, and in applied statistics, some probability level is considered good enough for the purpose of detecting batches of items containing more than a certain fraction of defectives. Where the acceptable probability is not attainable, it would be natural to take $p_i^h(x)$ as an aspect of choice. Cramér thus viewed "the probability that income will fall below some critical level as the criterion for ordering" the alternatives facing an insurance company (cited by Arrow 1951, p.423).

Writing $q_i^h(x) = \min\{p_i^h(x), p_i^{h*}\}$ and $q^h(x) = (q_1^h(x), q_2^h(x), \ldots)$, our assumption for the uncertainty case is that h has an L*-ordering given by q^h (instead of v^h under certainty).[5/] One will note the absence of any expected utility concept in this formulation. The expected value of $u_i^h(x, \sigma)$ is not well defined with u_i^h merely "ordinal", and it is unnecessary for determining a person's choice.

Putting q^h for v^h in Sections II to IV, the entire discussion there also applies to the case of uncertainty. Moreover, each person may have his own subjective probabilities regarding the possible states of nature. In the group decision problem discussed by Savage (1954, pp. 172-174), who assumed von Neumann-Morgenstern utility functions, the members of the group have different probability judgements but the same preference ordering on the set of possible outcomes. The latter restriction is of course not required in our formulation.

VI. Concluding Remarks

The solution of this paper to the arbitration problem is merely a repeated application of the idea of Pareto optimality to a multidimensional utility framework. If the solution is unique, it satisfies four conditions analogous to those of Nash which seem reasonable in the context, and it is the only solution that does so. Interestingly, it violates Arrow's independence of irrelevant alternatives condition although in no way does it depend on alternatives outside the admissible set. One could therefore argue that this Arrow condition is even more restrictive than has been thought.

The solution accomodates the case of uncertainty, n persons, and the group decision problem under uncertainty even when subjective probabilities are different, so it is quite versatile. All this is made possible by the basic assumption of lexicographic preferences, specifically, that we have L*-orderings. One reader's reaction to an earlier draft of this paper--having in view the familiar utility function--was that the assumption of L^*-orderings is "extreme". Most economists have become so accustomed to real valued utility functions that anything else is considered an aberration. It should be kept in mind that a utility function is an analytical tool pure and simple for the study of preferences, which are logically prior. "Utility, in its most general form, is ... represented by a finite or infinite dimensional vector with real components ... and these vectors are ordered lexicographically" (Chipman 1960, p. 221). Surely it would be a misapplication of Occam's razor not to use a more powerful if unfamiliar tool to solve existing problems, if the more familiar tool is inadequate.

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Notes

1. For masterly statements of the grounds for L^*-ordering and lexicographic utility in general, see Georgescu-Roegen (1954) and Chipman (1960). See also Day and Robinson (1973) for a useful continuity property of L^*-ordering under certain conditions, and Encarnación (1964a, b, c, 1965, 1969, 1983a, b) for L^*-ordering hypotheses in various contexts.

2. For convenience we will speak of $x \in A_i$ and $v_i(x) \in V_i(A_i)$ indifferently as v_i-optimal; identifying x with $v(x)$ should also cause no confusion.

3. A Pareto optimal point in a set is Pareto optimal in any of its subsets.

4. Also referred to as a weak ordering. R is a total ordering on X if for all $x, y \in X$, xRy or yRx, and for all $x, y, z \in X$, xRy and yRz implies xRz.

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