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LEXICOGRAPHIC GROUP CHOICE UNDER UNCERTAINTY

by

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Abstract

This paper proposes a solution to the problem of group decision under uncertainty when individuals have lexicographic preferences. The proposed solution satisfies four properties analogous to those that characterize the solution to the Nash bargaining problem, and if the set of feasible alternatives is fair in a certain sense, it is also the only solution that does so.

Lexicographic Group Choice Under Uncertainty*

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The usual approach to the problem of group choice under uncertainty is to assume von Neumann-Morgenstern utility functions (Savage 1954, Theil 1964). In this paper we will consider the problem under the basic assumption that individuals have lexicographic utility functions (see Fishburn 1974 for a survey of this literature, noting especially Georgescu-Roegen 1954 and Chipman 1960). The solution which we will propose satisfies four properties analogous to those that characterize the solution to the Nash (1950) bargaining problem. If the set of feasible alternatives is "fair" in a certain sense, it is also the only solution that does so. Section 1 defines an individual's lexicographic preferences. Section 2 describes a model of group choice for two persons and Section 3 states some properties of the solution. Section 4 is an extension to the n-person case. Section 5 considers the case of certainty and Section 6 is a concluding remark.

1. Lexicographic Preferences

Consider first the case of certainty. We assume multiple aspects or criteria of choice among alternatives, so that to each alternative x corresponds a vector $u(x) = (u_1(x), u_2(x), \ldots)$ where the "ordinal" utility function u_i ranks alternatives in terms of the ith choice criterion. Criterion u_i is more important or has higher priority than u_j if $i < j$. For each u_i there is a real number u_i^* denoting a critical or satisfactory level, so that if $u_i(x) \geq u_i^*$ one can say that x is acceptable with respect to the ith criterion.

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Writing $v_i(x) = \min\{u_i(x), u_i^*\}$ and $v(x) = (v_1(x), v_2(x), \ldots)$, x is preferred to y if and only if the first nonvanishing component of $v(x) - v(y)$ is positive, i.e. the preference ordering of the x's is given by the lexicographic ordering of the corresponding $v(x)$'s. An arbitrary positive monotonic transformation of u_i (which carries along u_i^*), hence of v_i $(i = 1, 2, \ldots)$, yields the same ordering.

In a previous note (Encarnación 1964) such a preference ordering was called an L^*-ordering in contrast to L-ordering where $u_i^* = \infty$ for all i. L^*-ordering is more flexible in permitting a less important criterion to play a role in choice once the criterion preceding it in the ranking is satisfied. Also, nothing prevents u_j, say, from being a function of $u_1, \ldots, u_{j-1}$ so that if the latter criteria are satisfied, u_j--a composite criterion so to speak--could become the maximand. Under L^*-ordering the common statement that wants are unlimited may be represented by an infinite-dimensional v and not in the usual way by the unboundedness of a real-valued utility function. Such a v also results by interpreting the first t_1 components as criteria of choice pertaining to events in time period 1, the next t_2 components as criteria for time period 2, and so on. Various interpretations thus make it natural to consider v as infinite-dimensional.

In the absence of certainty the outcome depends on the state of nature s, so we need to write $u_i(x, s)$ in place of $u_i(x)$. Let $S_i(x) = \{s \mid u_i(x, s) \geqq u_i^*\}$. Denoting the probability of s by $f(s)$ and writing

$$p_i(x) = \sum_{s \in S_i(x)} f(s) = \Pr\{u_i(x, s) \geqq u_i^*\}$$

let

$$q_i(x) = \min\{p_i(x), p_i^*\}$$

where p_i^* is the probability level considered acceptable for getting a satisfactory outcome in terms of the ith criterion, so that to each x corresponds $q(x) = (q_1(x), q_2(x), \ldots)$. The preference ordering of the x's is then given by the lexicographic ordering of the $q(x)$'s (cf. Encarnación 1965). The aim is to reach as many of the p_i^*'s as possible in their order of importance.

Using probabilities to rank alternatives is an old idea. Cramér considered "the probability that income will fall below some critical level as the criterion for ordering" the alternatives facing an insurance company (cited by Arrow 1951, p. 423). In the course of discussing various determinants of choice, Marschak (1950, p. 120, n. 10) has observed that "another important parameter is the probability that a certain variable (cash reserves, annual profit, etc.) fall below a constant, for example, below zero." Roy (1952) has argued in the same vein. The concept of a satisfactory probability is also familiar from the standard statistical practice of taking some probability level as good enough for the purpose of detecting batches of items containing more than a certain fraction of defectives. The classical Neyman-Pearson rule is similar; a specified probability of avoiding a type 1 error is taken as satisfactory. Closer to the present discussion, Charnes and Cooper (1965) have analyzed programming problems where the usual inequality constraints are only required to hold at stated probability levels.

In the certainty case, the aim is to reach as many u_i^*'s as possible, starting with the most important. Under uncertainty, one has to be satisfied with probabilities, and one speaks of acceptable risks in the pursuit of objectives.

2. The Model

Consider a group of two persons indexed by $k \in K = \{1, 2\}$. The notation of Section 1 is now to be understood to have superscripts to label different persons. Accordingly,

$$p_i^k(x) = \sum_{s \in S_i^k(x)} f^k(x) = Pr^k\{u_i^k(x, s) \geq u_i^{k*}\}$$

$$q_i^k(x) = \min\{p_i^k(x), p_i^{k*}\} \quad i = 1, 2, \ldots; \; k \in K$$

where u_i^1 is in general different from u_i^2 and probability judgments also differ. Person k's preference ordering of the x's is given by the lexicographic ordering of the $q^k(x)$'s, where $q^k(x) = (q_1^k(x), q_2^k(x), \ldots)$. Let γ_i^k be an arbitrary positive monotonic transformation, and write $r_i^k = \gamma_i^k(q_i^k)$. Clearly k's preference ordering is given just as well by the lexicographic ordering of the $r^k(x)$'s, where $r^k(x) = (r_1^k(x), r_2^k(x), \ldots)$.

A solution to the group choice problem selects a nonnull subset of the set of feasible alternatives, call the latter set A, as the group choice. Writing $r_i(x) = (r_i^1(x), r_i^2(x))$, an alternative $x \in A$ may be represented as a point $r(x) = (r_1(x), r_2(x), \ldots)$ in what may be called r-space. If B is a set of alternatives, we will write

$$\psi_i(B) = \{r_i(x) | x \in B\}, \quad \psi(B) = \{r(x) | x \in B\}.$$

Let $r_i^{k*} = \gamma_i^k(p_i^{k*})$. It is clear that $\psi(A)$ is bounded from above by the r_i^{k*}'s. We assume that $\psi(A)$ is a closed, connected, and noncountable set.

Let

$$A_i = \{x \in A_{i-1} \mid \forall y \in A_{i-1}: (\exists k: r_i^k(y) > r_i^k(x) \rightarrow \exists h: r_i^h(x) > r_i^h(y))\}, \quad i = 1, 2, \ldots,$$

where $A_0 = A$. A_1 is the set (nonnull because $\psi(A)$ is closed) of what may be called (Pareto) r_1-optimal elements in A. If $x \in A_1$, there is no y in A which everyone considers at least as good as x and someone considers better. The selection is narrowed by $A_2 \subset A_1$ whose elements are r_2-optimal as well as r_1-optimal, etc., and we propose

$$A^0 = A \cap A_1 \cap A_2 \cap \ldots$$

as the solution. Note that $A_i \subset A_{i-1}$ for all i and $A_j = A_{j-1}$ for a particular j only if the jth pair of criteria does not serve to narrow down either person's own choices. In general, A_i would be a proper subset of A_{i-1}.

We now assume that r^k is infinite-dimensional for all k, and we wish to show that $\psi(A^0)$ is a one-element set. Define a measure $\sigma_i = \sigma(\psi(A_i))$ of the "size" of $\psi(A_i)$ by

$$\sigma_i = \sum_{j=1}^{\infty} 2^{-j} \max_{(x, y)} \{d_j(x, y) \mid x, y \in A_i\}$$

where

$$d_j(x, y) = \left| \frac{r_j(x)}{|r_j(x)|} - \frac{r_j(y)}{|r_j(y)|} \right|$$

is a measure of distance between $r_j(x)$ and $r_j(y)$, and $|m| = (m \cdot m)^{1/2}$. Clearly $\sigma_i \leq \sigma_{i-1}$ for all i since A_i retains only those elements of A_{i-1}

that are r_i-optimal. The sequence $\{\sigma_i\}$ is thus a decreasing sequence and since $\sigma_i \geq 0$ for all i, it is bounded from below and therefore convergent. We make two assumptions: (i) $\sigma_j = \sigma_{j-1} > 0$ for at most a finite number of integers $j \in J$; (ii) there is a λ, $0 < \lambda < 1$, such that $\sigma_i \leq \lambda \sigma_{i-1}$ for all i not in J. Although (i) is stronger than is needed, it is hard to see why (i) should fail to hold in the nature of the case; (ii) also seems reasonable, for if $A_i \neq A_{i-1}$, A_i excludes most of the elements of A_{i-1} (viz. those which are not r_i-optimal). Under the assumptions the sequence $\{\sigma_i\}$ converges to 0, i.e. the sequence $\{\psi(A_i)\}$ converges to a point, so that we have the following

<u>Lemma.</u> $\psi(A^0)$ is a one-element set.

It is interesting that the assumption of infinite-dimensional utility vectors, which would seem to complicate matters, actually permits a considerable simplification of the problem. By a simple extension of Pareto optimality to the infinite-dimensional case, a unique solution in r-space results. We consider the solution A^0 fair in the sense that it is not arbitrary.

What might appear objectionable about the procedure is that we are letting at each stage a lower priority criterion decide, in effect, the choice with regard to a higher priority one. This seems to be precisely what one does, however, when he does not know how the alternatives rank in terms of the more important criterion. For example, in deciding where to spend a restful holiday in some new place, it may happen that one has no knowledge of the ranking of the possibilities as regards restfulness. There is no way then for him to decide on the basis of this criterion. But if he has a minor preference for the sea as against the mountains, he can narrow the field of choice by considering only places near the sea. The "arbiter" in a social choice problem is in a similar position for he has no way of selecting an alternative in A_{i-1} without being arbitrary, so he lets A_i narrow down the choice.

3. Properties of the Solution

Denoting the solution of A by $g(A)$, not necessarily the proposed solution A^0, consider the following properties that one might require of $g(A)$.

Condition 1 (invariance): The solution $g(A)$ is unchanged by arbitrary positive monotonic transformations of u_i^k and p_i^k $(i = 1, 2, \ldots; k \in K)$.

Condition 2 (symmetry): Suppose that $(a, b) \in \psi_i(A) \rightarrow (b, a) \in \psi_i(A)$ for all a, b, i. Then $\psi(g(A)) = \{\bar{r}\}$, say, where $\bar{r}^1 = \bar{r}^2$.

Condition 3 (Pareto optimality): No element of $g(A)$ is Pareto inferior to any element of A. (As usual, x is Pareto inferior to y if someone prefers y to x and no one prefers x to y.)

Condition 4 (rational choice): If $A \subset A'$ and $A \cap g(A') \neq \emptyset$, then $A \cap g(A') = g(A)$.

These conditions correspond to the four properties that characterize the Nash solution (cf. Luce and Raiffa 1957, pp. 126-127), though the underlying assumptions are of course different. Our Conditions 1 and 2 are called for by lexicographic utility functions while those of Nash are based on von Neumann-Morgenstern utility functions. Conditions 3 and 4 are the same as those of Nash. Condition 3 has strong traditional appeal, and Condition 4 is Arrow's requirement for a "rational" choice function (Arrow 1959, Definition C4).

Theorem 1. If $g(A) = A^0$, $g(A)$ satisfies Conditions 1 to 4.

Proof. A_i is unaffected by a positive monotonic transformation of u_i^k or of p_i^k, which carry along u_i^{k*} and p_i^{k*} respectively $(i = 1, 2, \ldots)$, so A^0 remains the same and Condition 1 is satisfied. Given the hypothesis of Condition 2, $\psi_i(A)$ is symmetrical with respect to the diagonal line $r_i^1 = r_i^2$, and therefore under the procedure that determines A^0, $\psi_i(A_i)$ is symmetrical with respect to the same line $(i = 1, 2, \ldots)$. Since $\psi(A)$ is connected, in view of the Lemma we must then have $\psi(A^0) = \bar{r}$, where $\bar{r}^1 = \bar{r}^2$, so that Condition 2 holds. Condition 3 is clear since x is Pareto inferior to some y in A only if x does not belong to A_i for some i, but $x \varepsilon A^0$ implies $x \varepsilon A_i$ for all i. To show Condition 4 we first prove that its hypothesis implies (i) $A \cap g(A') \subset g(A)$. Statement (i) is false only if there is a z such that (ii) $z \varepsilon A \cap g(A')$ but (iii) $z \varepsilon A - g(A)$. Given that $g(A) = A^0$, statement (iii) implies a least integer j such that $\sim z \varepsilon A_j$, using $\sim$ to denote negation. But $\sim z \varepsilon A_j$ implies $\sim z \varepsilon A_j'$ since z cannot be r_j-optimal in A' if it is not r_j-optimal in the smaller set A. Hence $\sim z \varepsilon g(A')$, which contradicts (ii), so that (i) holds. Therefore under the hypothesis of Condition 4 which says that $A \cap g(A')$ is nonnull, $\psi(A \cap g(A')) = \psi(g(A))$ from (i) and the Lemma. Hence $y \varepsilon g(A)$ implies $y \varepsilon A \cap g(A')$, which establishes Condition 4 and completes the proof.

We have described the solution A^0 as fair, but it is possible that A itself might in some sense be unfair to begin with. Call r normalized if $r_i^{k*} = 1$ for all i, k.

<u>Definition 1.</u> A_i is unfair if $\forall x: (x \in A_i \rightarrow (\exists k: r_i^k(x) = 1$ & $\exists h: r_i^h(x) < 1))$, with r normalized; otherwise, A_i is fair. If A_i is fair for all i, A is fair; otherwise, A is unfair.

This is a relatively weak requirement on fairness of the feasible set since it only rules out the case where some person necessarily satisfies his ith criterion while another person does not (in the solution A^0). If A_i is fair, there is an $r_i \in \psi_i(A_i)$ with $r_i^k = 1$ for all k or else $r_i^k < 1$ for all k. This does not exclude the possibility of an $x \in A_i$ such that $\exists k$: $r_i^k(x) = 1$ & $\exists h: r_i^h(x) < 1$.

<u>Theorem 2.</u> A is fair if and only if there is a normalized r such that $x \in g(A) = A^0 \rightarrow r^1(x) = r^2(x)$.

<u>Proof.</u> The "if" part is obvious. To show the "only if" part, suppose A is fair and r is normalized. It is clear that if $\psi_i(A^0) = \{\hat{r}_i\}$ has $\hat{r}_i^k < 1$ for all k, we can always put $\hat{r}_i^1 = \hat{r}_i^2$ by using Condition 1. If $\hat{r}_i^k = 1$ for all k, there is no problem. The only case that needs examination has $\hat{r}_i^k = 1$ and $\hat{r}_i^h < 1$ for some k, h, i despite the fact that, since A_i is fair, there exists $r_i \in \psi_i(A_i)$ with $r_i^k < 1$ for all k. If such a case holds for A^0, we will say that $\psi(A^0)$ is a corner point. Suppose, then, that $\psi(A^0)$ is corner point, and let $B = A - A^0$. If $\psi(B^0)$ is also a corner point, let $C = B - B^0$, and so on. Since $\psi(A^0), \psi(B^0), \ldots,$ would be one-element sets, $\psi(A)$ cannot be exhausted by removing a countable number of its elements, so we must have a $\psi(C^0)$, say, which is not a corner point. By Condition 1 we can put $\psi(C^0) = \{\ddot{r}\}$ where $\ddot{r}^1 = \ddot{r}^2$. Consider D satisfying the hypothesis of Condition 2 such that $C \subset A \subset D$ and for each i, the boundary points of $\psi_i(C)$ and $\psi_i(D)$ on the diagonal line $r_i^1 = r_i^2$ are

identical. By Condition 2, $\psi(D^0) = \{\bar{r}\}$ with $\bar{r}^1 = \bar{r}^2$. Since every point r satisfying $r^1 = r^2$ and belonging to $\psi(D)$ also belongs to $\psi(C)$, we have $C \cap D^0 \neq \emptyset$ and therefore $C \cap D^0 = C^0$ by Condition 4. Hence $\psi(C^0) = \psi(D^0)$, or $\{\bar{r}\} = \{\tilde{r}\}$, so that $\psi(A^0) = \{\bar{r}\}$ since $C \subset A \subset D$. In other words $\psi(A^0)$ cannot be a corner point, proving the theorem.

Theorem 2 implies that if A is fair, the solution A^0 will not let any person k satisfy his ith criterion if some h does not satisfy his. Either everyone satisfies his ith criterion or everyone does not, and this holds for all i. The solution A^0 thus seems fair also in an ethical sense.

<u>Theorem 3</u>. If A is fair and $g(A)$ satisfies Condition 1 to 4, $g(A) = A^0$.

<u>Proof</u>. Let the hypothesis hold. By Theorem 2 we can choose r so that $\psi(A^0) = \{\hat{r}\}$ with $\hat{r}^1 = \hat{r}^2$, and by Condition 1 $g(A)$ would be unchanged. We need to show that $\psi(g(A)) = \{\hat{r}\}$. As in the proof of Theorem 2 consider A' satisfying the hypothesis of Condition 2 such that $A \subset A'$ and for each i, the boundary points of $\psi_i(A)$ and $\psi_i(A')$ on the diagonal line $r_i^1 = r_i^2$ are identical. By Condition 2, $\psi(g(A')) = \{\bar{r}\}$ with $\bar{r}^1 = \bar{r}^2$. Since every point satisfying $r^1 = r^2$ and belonging to $\psi(A')$ also belongs to $\psi(A)$, $A \cap g(A') \neq \emptyset$ and therefore $A \cap g(A') = g(A)$ by Condition 4. Hence $\psi(g(A)) = \{\bar{r}\}$. Let $x \in g(A)$ and $y \in A^0$ so that $r_1(x) = \bar{r}_1$ and $r_1(y) = \hat{r}_1$. Since y is r_1-optimal, $\bar{r}_1 > \hat{r}_1$ is not possible. If $\bar{r}_1 < \hat{r}_1$, every person k considers y better than x in terms of r_1^k, in which case $g(A)$ violates Condition 3. Hence $\bar{r}_1 = \hat{r}_1$. With this fact in hand, the argument can be repeated with respect to r_2 to get $\bar{r}_2 = \hat{r}_2$, etc., so that $\bar{r} = \hat{r}$ as required.

The following thus holds from Theorems 1 and 3.

<u>Theorem 4</u>. If A is fair, $g(A) = A^0$ if and only if $g(A)$ satisfies Conditions 1 to 4.

4. Generalization to n Persons

A review of Section 2 will show that the procedure for determining A^0 is in no way dependent on the fact that there are only two persons in the group. We can replace $K = \{1, 2\}$ by $K = \{1, \ldots, n\}$, write $r_i(x) = (r_i^1(x), \ldots, r_i^n(x))$, and the appropriate discussion of the choice problem for n persons would be the same almost word for word.

A review of Section 3 will also show that it applies to n persons after modifying Condition 2, which would now read as follows.

<u>Condition 2' (n-person symmetry)</u>: Suppose that $c \in \psi_i(A) \rightarrow \pi(c) \in \psi_i(A)$ for all i, c and $\pi(c)$, where $\pi(c)$ is a permutation of the components of $c = (c^1, \ldots, c^n)$. Then $\psi(g(A)) = \{\bar{r}\}$ with $\bar{r}^k = \bar{r}^h$ for all $k, h \in K$.

With this revision, $\psi_i(A)$ is symmetrical with respect to the hyperline $r_i^1 = \ldots r_i^n$. The definition of a fair A remains the same, the discussion would be applicable to n persons almost word for word, and we would have the same theorems for n persons.

5. The Case of Certainty

In this case the concept of satisfactory probability levels becomes vacuous and it would suffice to say that person k's preference ordering of the x's is given by the lexicographic ordering of the corresponding $v^k(x)$'s (see the first paragraph of Section 1). Interpreting $r_i^k = \gamma_i^k(q_i^k)$ now as $r_i^k = \gamma_i^k(v_i^k)$, with r normalized by $\gamma_i^k(u_i^{k*}) = 1$ for all i and k, a review of the preceding sections will show that the entire discussion (ignoring references to probabilities) applies as well as to the certainty case. All proofs are the same and the same theorems hold.

Thus we have a possible solution to the social choice problem under certainty and under uncertainty. We have described the solution A^0 as fair in the sense of not being arbitrary, and it also seems fair in an ethical sense. This suggests the possibility of making welfare comparisons.

Definition 2. In situation x, person 1 is better off than 2 if the first nonvanishing difference $w_i^1(x) - w_i^2(x)$, $i = 1, 2, \ldots,$ is positive, where $w_i^k(x) = 1$ if $r_i^k(x) = 1$, $w_i^k(x) = 0$ otherwise, and r is normalized.

There is here an interpersonal comparison of utilities made possible by the fact that under L^*-ordering, each person's criteria of choice are ordered by importance so that different persons' ith criteria (even if not the same) can be matched. What only counts, however, is whether or not criteria are satisfied, not the degrees to which they are failed, which are noncomparable. From Definition 2 and the n-person version of Theorem 2 follows directly

Theorem 5. A is fair if and only if no one is $x \in A^0$ is better off than anyone else.

As a solution to the social choice problem, A^0 necessarily violates one of the Arrow (1963) requirements for a social choice function. Let society's and k's orderings on the set of conceivable alternatives be denoted by R and R^k respectively. R^k is a total (i.e. weak) ordering, and xR^ky if and only if $\sim yP^kx$, P^k being k's preference relation. R, implicitly defined in

$$g(A) = \{x \in A \mid \forall y \in A: xRy\},$$

is a total ordering if $g(A)$ satisfies Condition 4 (see Arrow 1959, Theorem 3). Hence with $g(A) = A^0$, R is also a total ordering, and x is socially preferred to y (or xPy) if and only if $\sim yRx$.

Murakami's (1961) formulation of the Arrow impossibility theorem shows that R cannot simultaneously satisfy the Pareto principle and the free triple, nondictatorship and independence of irrelevant alternatives (IIR) conditions. The last condition says the following (R_A is the social ordering on A and R_A^k is k's).

IIR: R_A is invariant with respect to any changes in $\{R^k\} = \{R^1, \ldots, R^n\}$ that do not affect $\{R_A^k\} = (R_A^1, \ldots, R_A^n)$.

The first three requirements are easily satisfied in our model, so IIR must be violated. Consider a 2-person society and $x, y \in A$ such that $r_1^1(x) > r_1^1(y)$ and $r_1^2(y) > r_1^2(x)$, so that xP^1y and yP^2x. With $r_2(x) > r_2(y)$, xPy. Suppose now that each k lowers his u_2^{k*} level so that $r_2^k(x) = r_2^k(y) = 1$ in the new normalization. Individual preferences are unaffected, but now, r_2 no longer discriminates between x and y so that yPx if $r_3(y) > r_3(x)$. Thus IIR is violated not because R_A depends on alternatives outside A -- the label "independence of irrelevant alternatives" is misleading in this regard--but because R_A in our model depends on the individual parameters of choice (the u_i^{k*}'s) and the same $\{R_A^k\}$ could result from different sets of parameters (cf. Encarnación 1969).

6. Concluding Remark

The proposed solution to the group choice problem satisfies Arrow's requirement for a rational choice function, Pareto optimality, symmetry, and an invariance property. It is also the only solution that satisfies these properties if the feasible set is fair in a certain sense. We consider the solution fair in not being arbitrary and also fair from an ethical viewpoint. As a solution to the problem of social choice, it perforce violates Arrow's "independence of irrelevant alternatives" condition which however, in our view, is the least important of the Arrow conditions for a social choice function.

One aspect of the model of this paper remains to be noted explicitly. If each component of the utility vector pertains to a particular time period, we would in general expect later-time components to have higher-numbered subscripts. The lower value of the future has then to do not with discounting but with the lower priority attached to later events in a lexicographic utility framework. There is no need for individual or social rates of time preference, which are used only to convert time series into real numbers.

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