BAUMOL AND TOBIN ON THE TRANSACTIONS DEMAND FOR CASH: A PEDAGOGICAL NOTE

by

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It seems likely that the papers by Baumol and Tobin dealing with the relationship between the interest rate and the transactions demand for cash are on most reading lists for courses in macro theory and monetary economics. Students have probably puzzled over the objective function used by Baumol in his model, in view of Tobin's remark that "Baumol's calculation of interest cost is rather difficult to understand" on the ground that Baumol makes interest cost proportional to the average cash balance. This would seem to imply, according to Tobin, that interest cost would be zero in the situation where cash is withdrawn from investment only at the moment when it is needed, but this would entail infinitely high brokerage costs, so that "it hardly seems a logical zero from which to measure interest costs." Tobin's own formulation of the problem is to maximize interest earnings net of brokerage costs. Interestingly enough, though not really surprisingly, using Tobin's objective function in Baumol's framework gives the same formula as Baumol's.

Following Baumol, let $T$ be the amount of dollars which is to be paid out uniformly over a unit period of time, $b + kC$ the broker's fee when $C$ dollars are withdrawn from investment, and $i$ the rate of interest. The number of withdrawals is $n = T/C$ and, following Tobin, we wish to choose $C$ so as to maximize interest earnings net of total broker's fees $(b + kC)n$. Figure 1 depicts the situation.
Each withdrawal \( C \) suffices to finance pay-out requirements for \( \frac{1}{n} \) of the period. During the first interval, the invested funds \((n - 1)C\) earn interest equal to \((n - 1)Ci/n\); during the second interval, interest earnings amount to \((n - 2)Ci/n\), etc. Total revenue \( R \) thus amounts to

\[
R = (n - 1)Ci/n + (n - 2)Ci/n + \ldots + 2Ci/n + Ci/n
\]

\[
= (n - 1)Ci/2 = (T - C)i/2
\]

Net revenue \( NR \) is therefore

\[
NR = (T - C)i/2 - (b + kC)T/C \quad \text{and maximizing this with respect to } C \quad \text{gives Baumol's formula}
\]

\[
C = (2bT/i)^{1/2}
\]
We see that since the interest earned is \((T - C)i/2\), the average amount invested must be \((T - C)/2\). This fact could have been obtained directly from observing that because of the uniform pay-out, the average balance (including funds invested and the cash balance) is \(T/2\) while the average cash balance is \(C/2\).

In Baumol's formulation of the problem, the objective was to minimize

\[(b + kC)T/C + Ci/2\]

where \(Ci/2\) is what Baumol calls the interest cost of holding cash.

It is clear why we get the same formula as Baumol's, as minimizing this expression is equivalent to maximizing \(MR\) since \(Ti/2\) is a constant.

REFERENCES

