Institute of Economic Development and Research
SCHOOL OF ECONOMICS
University of the Philippines

Discussion Paper No. 71-20 September 8, 1971

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In a recent paper [1] P.K. Bardhan and T.N. Srinivasan have presented a simple general equilibrium model of share tenant and landlord agricultural production. Their contribution is notable as a primary attempt to provide an analysis of sharecropping in a general equilibrium framework, as opposed to its usual partial equilibrium formulation. S.N.S. Cheung [2] has also attempted to analyze cropsharing tenancy in a general equilibrium setting, but his model is much less general than that presented by Bardhan and Srinivasan. In addition, the authors contribute to the theory of firm behavior under technological uncertainty by assuming risk aversion, in contrast to the usual assumption of risk neutrality.

The discussion by Bardhan and Srinivasan of the properties of the model for a number of cases is interesting and useful. However, their account of production in the presence of uncertainty is brief and not very careful. For example, Bardhan and Srinivasan incorrectly derive the

*I am grateful to Jose Encarnacion, Jr. for encouragement and helpful suggestions.

necessary conditions for tenant and landlord maximization of expected utility. This, of course, leads them to an incorrect formulation of their comparative static results. The purpose of this note is to clarify the analysis of production under uncertainty given by Bardhan and Srinivasan. First we derive the appropriate necessary conditions. Then we present the modification of the comparative static result which is required.

We shall consider the input choices of the tenant alone since only partial equilibrium results are presented by the authors. The modification required on the landlord side is similar. The tenant is assumed to maximize the expected utility of consumption (ignoring the superscripts used by Bardhan and Srinivasan)

$$E\left[U(C)\right] \tag{1}$$

subject to the constraint

$$C = (1 - r)AF(H, \ell) + w(1 - \ell) \tag{2}$$

where r is the share of output paid as rent to the landlord, A is a multiplicative random uncertainty parameter, F is the production function, H is the amount of land utilized by the tenant, ℓ is the amount of labor the tenant devotes to his farm, $(1 - \ell)$ is the amount of labor devoted to wage earning occupations, and w is the given wage rate. Both U and F are assumed to be strictly concave functions. Note that, although Bardhan and Srinivasan do not explicitly make such a specification, A must reasonably be assumed to be a positive random variable, since negative output at positive input levels in agriculture is not possible.

The necessary conditions for the tenant's interior maximum given by the authors are

$$F_1 = 0 \tag{83}$$

$$E\left[U'(C)((1 - r)AF_2 - w)\right] = 0 \tag{84}$$

We will argue that condition (83) is not correct. But first we note an implication of (83). (This consideration also applies to Bardhan and Srinivasan's analysis in the case of certainty.) If the production function is strictly increasing in H and ℓ, then (83) cannot be satisfied at a finite level of H. However, we can show that the appropriate condition corresponding to (83) implies that F_1 minus a risk term is equated to zero. Hence, this condition is satisfied at a finite level of H, even for a production function which is strictly increasing in the inputs.

Differentiating expected utility with respect to H and equating to zero we have

$$(3) \qquad E\left[U'(C)(1 - r)AF_1\right] = 0$$

This may be written as

$$(4) \qquad F_1 + \frac{Cov(U'(C),(1 - r)AF_1)}{(1 - r)E(A)E\left[U'(C)\right]} = 0$$

where Cov denotes the covariance. Now Bardhan and Srinivasan's specification in (83) implies that the second term on the left hand side of (4) is zero. This in turn implies that $Cov(U'(C),(1 - r)AF_1)$ is equal to zero. If the tenant utility function were linear, then $U'(C)$ would be constant, and the covariance would vanish. However, Bardhan and Srinivasan postulate a strictly concave utility function. We can show that the covariance is negative for strictly concave utility functions.

$Cov(U'(C),(1 - r)AF_1)$ may be written as

$$(5) \quad (1 - r)F_1E\left[U'(C)(A - EA)\right] - (1 - r)F_1E\left[U'(C)\right]E\left[A - EA\right].$$

The second term is equal to zero, and it is easy to show that the first term is negative in the case of risk aversion. Clearly for U strictly concave $(U''(C)<0)$

$$U'(C)<U'(EC) \tag{6}$$

if $A>EA$, since C is a linear function of A. Multiplying by $(A - EA)$

$$U'(C)(A - EA)<U'(EC)(A - EA) \tag{7}$$

if $A>EA$. This inequality also holds for $A<EA$ since the inequality sign in (6) is reversed but negative $(A - EA)$ causes an offsetting reversal of the inequality sign. Taking expectations on both sides of (7) we get

$$E\left[U'(C)(A - EA)\right]<U'(EC)E\left[A - EA\right]. \tag{8}$$

The right hand side is equal to zero, and therefore the left hand side is negative. Hence, the first term in (5) is negative and $Cov(U'(C),(1 - r)AF_1)$ is also negative. The risk averter equates F_1 minus a term due to risk aversion to zero. We conclude therefore, that the specification given by Bardhan and Srinivasan in (83) is incorrect.

Now we turn to the comparative static result reported by Bardhan and Srinivasan. First we must change the elements in the Jacobian matrix used by the authors. The Jacobian matrix of equations (3) and (84) has the following elements:

$$a_{11} = E\left[U'(C)(1 - r)AF_{11} + U''(C)((1 - r)AF_1)^2\right]$$

$$a_{12} = E\left[U'(C)(1 - r)AF_{12} + U''(C)(1 - r)AF_1((1 - r)AF_2 - w)\right]$$

$$a_{21} = E\left[U'(C)(1 - r)AF_{21} + U''(C)(1 - r)AF_1((1 - r)AF_2 - w)\right]$$

$$a_{22} = E\left[U'(C)(1 - r)AF_{22} + U''(C)((1 - r)AF_2 - w)^2\right]$$

The Jacobian is positive since a monotone increasing concave function of a concave function is concave.

Bardhan and Srinivasan define A as

$$A = \alpha u + \beta$$

where u is a random variable and α and β are constants. The authors examine the effect on input choice of an increase in pure uncertainty by differentiating the necessary conditions with respect to α and allowing β to vary so that

$$\frac{d\beta}{d\alpha} = -E(u).$$

Differentiating conditions (3) and (84) yields

$$\left[a_{ij}\right]\begin{bmatrix}\frac{dH}{d\alpha}\\ \frac{d\ell}{d\alpha}\end{bmatrix} = \begin{bmatrix}-(1 - r)E\left[\left\{U'(C)F_1 + U''(C)(1 - r)AF_1F\right\}(u - Eu)\right]\\ -(1 - r)E\left[\left\{U'(C)F_2 + U''(C)((1 - r)AF_2-w)F\right\}(u - Eu)\right]\end{bmatrix}$$

The upper element on the right hand side of our equation (9) is zero in equation (87) of the paper under discussion. Bardhan and Srinivasan show in footnote 12 of their paper that the lower element on the right hand side of (9) is positive under the assumption of constant <u>relative</u> risk aversion. Using their argument it is possible to show that the upper element on the right hand side of (9) is also positive.

However, it is interesting to note that it is also possible to show that the assumption of constant <u>absolute</u> risk aversion is sufficient for both terms on the right hand side of (9) to be positive. The assumption that the index of absolute risk aversion, $a(C) = -U''(C)/U'(C)$, is constant implies that the measure of relative risk aversion, $r(C) = -U''(C)\cdot C/U'(C)$, is an increasing function of C. From the argument used above we know that $(1 - r)F_2 E\left[U'(C)(A - EA)\right] < 0.$[1] Therefore we need to show that

$$(10) \qquad (1 - r)E\left[U''(C)((1 - r)AF_2 - w)F(A - EA)\right] < 0.$$

The right hand side of (10) may be written as

$$(11) \quad E\left[U''(C)C((1 - r)AF_2 - w)\right] - E\left[U''(C)((1 - r)AF_2 - w)E(C)\right].$$

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[1]The authors assume $\alpha > 0$, so $(u - E(u))$ has the same sign as $(A - E(A))$.

Let us define $\bar{C}$ as that value of C for which $(1 - r)AF_2-w = 0$. If the index of relative risk aversion is increasing

$$(12) \qquad -\frac{U''(C)\cdot C}{U'(C)} > r(\bar{C})$$

for $(1 - r)AF_2-w>0$. Multiplying by $-U'(C)((1 - r)AF_2-w)$

we have

$$(13) \qquad U''(C)C((1 - r)AF_2-w) < -r(\bar{C})U'(C)((1 - r)AF_2-w).$$

This inequality holds for all A since for $(1 - r)AF_2-w<0$ there are two offsetting reversals of the inequality sign. Taking expectations on both sides of (13) yields

$$(14) \quad E\left[U''(C)C((1 - r)AF_2-w)\right] < -r(\bar{C})E\left[U'(C)((1 - r)AF_2-w)\right].$$

The right hand side is equal to zero by the necessary condition (8b). Hence the left hand side is negative. By a similar argument we can show that constant absolute risk aversion implies

$$(15) \quad E\left[U''(C)((1 - r)AF_2-w)\right] = -a(\bar{C})E\left[U'(C)((1 - r)AF_2-w)\right] = 0.$$

It follows, therefore, that (10) is negative and the lower element on the right hand side of (9) is positive. In like manner, the upper element can also be shown to be positi on the assumption of constant absolute risk aversion.

Now solving for $dH/d\alpha$ we obtain

(16) $$\frac{dH}{d\alpha} = -(1 - r)a_{22}\frac{1}{|J|}E\left[\left\{U'(C)F_1 + U''(C)(1 - r)AF_1F\right\}(u - Eu)\right]$$

$$+(1 - r)a_{12}\frac{1}{|J|}E\left[\left\{U'(C)F_2 + U''(C)((1 - r)AF_2 - w)F\right\}(u - Eu)\right]$$

where $|J|$ is the Jacobian determinant. We find that the assumptions of Bardhan and Srinivasan do not guarantee the sign of $dH/d\alpha$, since the sign of a_{12} is unknown. However, we may take the sign of a_{12} as defining H and ℓ as complementary of substitute inputs according as a_{12} is positive or negative.[2] When complementarity exists between H and ℓ, an increase in the wage rate will (under the assumption of constant or decreasing absolute risk aversion) lead to a reduction in the optimal level of use of both factors. Hence, if land and labor are complementary inputs, an increase in uncertainty concerning production implies that the tenant will lease in less land at optimum.

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[2]This is analogous, but not equivalent, to the assumption of positive cross-partial derivatives on the production function for production under certainty.

REFERENCES

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